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ROYAL AUSTRALIAN AIR FORCE

AIR TRAINING CORPS

Manual For Cadets

(GROUND DUTIES)

Issued for the information and guidance of all concerned.

By Command of the Air Board,

E. J. Mulrooney

Secretary.

AIR FORCE HEAD-QUARTERS,
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PRELIMINARY COURSE

ALL CADETS

1 Mathematics **1A/1 Arithmetic**

C.3969/42—1.

1.—MATHEMATICS.

1. Introduction.

A mistake in an elementary calculation may, on occasions, be the cause of serious trouble to the crew of an aircraft—trouble that could have been avoided, had the member responsible not become "rusty" in a process he was once able to perform readily.

In every branch of technical training in the Royal Australian Air Force, success depends on mental alertness, and, in many subjects, a thorough grasp of the basic processes of Mathematics.

The preliminary course in Mathematics contains the fundamentals of Arithmetic, Algebra and Geometry. It has been drawn up so as to stimulate mental alertness, to impart the necessary information and to provide practice in setting out work in an orderly manner.

In Mathematics, the importance of the orderly arrangement of calculations cannot be too strongly stressed. It permits work to be quickly checked and so enables errors to be readily detected. Constant practice in the habit of tidiness will result in the achievement of accuracy, with speed, in all calculations.

1. PLACE VALUE OF A FIGURE.

Before dealing with the fundamental processes, something must be said of the **place value** of a figure.

When we write down the number 1276, we are really employing a short method of writing :—

$1000 + 200 + 70 + 6.$

If we rearrange the same figures to form the number 6721, the figures take on new values, due to their altered position. In their new positions they express the sum of :—

$6000 + 700 + 20 + 1.$

We see, therefore, that in any number containing more than one figure, **each figure has a value which depends solely upon its position or its place in the sequence of figures.**

The last figure of a **whole number** always represents the units place. For each place we shift it to the **left** of the units position, we increase its value 10 times, or in other words, a figure always has a value one-tenth of what it would have, if it were in the place immediately to its left.

In the whole number 15, the **place** value of the figure 5 is 5 units ; in 51, it is 50, i.e., 5 x 10 ; in 510, it is 500, i.e., 5 x 100 or 50 x 10, and so on.

Here we have the first example of the importance of **orderly arrangement,** since we have seen that figures take on a value according to their place in **any arrangement** of figures.

2. ADDITION.

The symbol +, when placed between two numbers, indicates that they are to be added.

Suppose we were asked to find the sum of the following numbers :—7543, 21, 696, 205, 10301. In setting out these numbers for addition, we must be careful to arrange the numbers so that all figures having the same place value are in the same column, thus :—

$$\begin{array}{r} 7543 \\ 21 \\ 696 \\ 205 \\ 10301 \\ \hline 18766 \\ \hline \end{array}$$

Process.—The sum of the right hand column is 16 units, i.e., one 10 and 6 units. Hence 6 is put down in the units column and 1 is added to the tens column, and so on.

2. Addition—*continued.*

Instead of being arranged in this manner, numbers to be added are frequently set down in a row across the page and connected by the + symbol.

For example, find the sum of the following numbers :—

123, 49, 7204 and 7.

These are set down thus :—

123 + 49 + 7204 + 7 and the answer is 7383.

In such cases, we must determine the place value by inspection and add all the figures having the same place value.

This example should be checked by setting out as in the first example.

EXERCISES.

Find the sum of the following sets of numbers by re-arranging them one under the other, according to place value :—

(*a*) 55452, 3837, 76996, 121, 47825.

(*b*) 28, 76, 126, 92, 75, 418, 9, 3851, 101, 27341.

2. Find the sum of the following sets of numbers by inspection :—

(*a*) 37949, 75889, 86945, 7978.

(*b*) 25, 5, 125, 80, 6478, 12.

3. From the following table read off and write down the total of the numbers in each line across the page, and also the total for each vertical column.

	Column 1.	Column 2.	Column 3.	Column 4.
Line *A* ..	543	27	1647	8
Line *B* ..	2464	153	24	95
Line *C*	27	9	106	170
Line *D* ..	501	94	2386	15

Add the totals of the four columns together and similarly add the totals of the four lines. What do you observe about the answers ?

3. SUBTRACTION.

Subtraction is the process of finding how much greater one number or group of numbers is than another number or group of numbers.

The excess is called the **Remainder** or **Difference.**

The symbol —, placed between two numbers, indicates that the second is to be subtracted from the first.

Thus, suppose we are asked to find how much greater 73409 is than 18254. This may be set out as in the first example in addition, care being taken to set down figures that have the same **place value** in the same column thus:—

$$\begin{array}{r} 73409 \\ 18254 \\ \hline 55155 \\ \hline \end{array}$$

This may also be performed by inspection :—

$73409 - 18254 = 55155.$

Addition and subtraction are often combined. For example —

$1046 + 732 - 846 + 7483 - 2386.$

The problem really means :—From 1046, 732 and 7483 taken together, subtract both 846 and 2386. There are really three steps involved in obtaining the answer :—

1st Step—	**2nd Step—**	**3rd Step—**
1046	846	9261
732	2386	− 3232
7483	3232	6029
9261		

Answer : 6029.

The case where the quantity to be subtracted is greater than the quantity from which it is to be subtracted will be considered in the Algebra section of this course.

EXERCISES.

1. In the following exercises, set the numbers one under the other before subtracting :—

   (*a*) 93486 — 7089 ; (*b*) 71001 — 9087.

2. Perform the following subtractions by inspection :—

   (*a*) 1597 — 609 ; (*b*) 85002 — 407.

3. Simplify the following :—

   (*a*) 9847 — 3848 + 741 — 2145 — 87.

   (*b*) 38452 — 17634 — 3841 + 246 + 1837.

Exercises—*continued.*

4. An author claims that Britain and France intercepted, by the end of November, 1939, a total of 500,000 tons of contraband destined for Germany. The following is an extract from the list detailing the haul:—

> Oilseeds, 21,100 tons; phosphates, 31,100 tons; fibres, 4,300 tons; jute, 1,500 tons; wool, 460 tons; cotton, 5,400 tons; hides and skins, 2,400 tons; petroleum products, 87,540 tons; iron ore, 81,500 tons; aluminium and bauxite, 21,500 tons.

How many tons are unaccounted for on this list?

4. MULTIPLICATION.

Multiplication is a concise form of repeated addition and is indicated by the symbol $\times$. For example, 6 multiplied by 4 may be written:—$6 \times 4 = 6 + 6 + 6 + 6 = 24$. The number to be multiplied (i.e., 6) is called the **MULTIPLICAND,** the number by which it is to be multiplied (i.e., 4) is called the **MULTIPLIER,** and the result (i.e., 24) is called the **PRODUCT.** The multiplicand and the multiplier are often called the **FACTORS** of the product. As another example, multiply 744 by 285. As we have seen, 285 is really $200 + 80 + 5$. We should obtain the desired answer by multiplying 741 by 200, 80 and 5 in turn, and adding the results together.

$$
\begin{array}{r@{\;}c@{\;}r}
741 & & \\
285 & & \\
\hline
148200 & = & 200 \times 741 \\
59280 & = & 80 \times 741 \\
3705 & = & 5 \times 741 \\
\hline
211185 & = & 285 \times 741 \\
\hline
\end{array}
$$

It is unnecessary for us to set the calculation out as fully as this if we keep in mind the position in which the figures should be placed.

It is enough to remember to place the first figure obtained in each multiplication directly under the multiplying figure.

$$
\begin{array}{r}
741 \\
285 \\
\hline
1482 \\
5928 \\
3705 \\
\hline
211185 \\
\hline
\end{array}
$$

Notice that 741×285 gives the same result as 285×741; that is, the order in which the factors are multiplied does not affect the result.

4. **Multiplication**—*continued.*

Multiplication may be done mentally, i.e., by inspection, provided you remember the place values of the figures in the factors.

For example, multiply 26 by 17.
$26 \times 17 = 442.$

The mental process is as follows :—

20 multiplied by 10 = 200.
6 „ „ 10 = 60 and 200 = 260.
20 „ „ 7 = 140 and 260 = 400.
6 „ „ 7 = 42 and 400 = **442.**

The answer **442** is the only part of the working that is written down.

This process may seem involved but is extremely easy after a little practice.

In performing a multiplication always choose as the multiplier the number which is easier to handle. This is generally the number which has the least number of figures. For example, in calculating 247×98795, take 247 as the multiplier.

NOTE.—If you have not been in the habit of multiplying from the left, i.e., starting with the first figure from the left in the multiplier, you are definitely advised to acquire the habit.

In any example of the following type, in which there is a mixture of addition, subtraction and multiplication, the multiplication must be performed first :—

$$\begin{aligned} 12 \times 29 + 36 - 2 \times 47 &= 348 + 36 - 94 \\ &= 384 - 94 \\ &= \mathbf{290.} \end{aligned}$$

Where there are brackets, as in the following example, the process **within the brackets** must be performed first :—

$$\begin{aligned} 12 \times (29 + 36) - 2 \times 47 &= 12 \times 65 - 94 \\ &= 780 - 94 \\ &= \mathbf{686.} \end{aligned}$$

By comparing this example with the former one, it can readily be seen that the correct treatment of the brackets made all the difference to the answer.

EXERCISES.

(For practice in speed and accuracy, allow yourself 5 minutes for Exercises 1 and 2 together.)

1. Perform the following multiplications mentally and write the answers in your book :—

(*a*) 273×12; (*b*) 874×7; (c) 903×11; (*d*) 211×20; (*e*) 18×15.

Exercises—*continued.*

2. Set out the following exercises in an orderly fashion :—

(*a*) 763×846; (*b*) $31 \times 53 \times 29$; (*c*) $38 \times 23 + 23 \times 24 - 23 \times 17$; (*d*) $18 \times (26 - 7) \times 3 - 34 \times 18 + 19$.

3. (*a*) A cubic foot of aluminium is said to weigh 165 lb., whereas a cubic foot of lead weighs 711 lb. By mental calculation, how many lb. would 8 cubic feet of each metal weigh ?

(*b*) Taking the distance travelled by a point on the Equator in one revolution of the Earth as 24,900 miles, how far would it travel for 365 revolutions ?

5. DIVISION.

Division is a concise form of repeated subtraction and is indicated by the symbol $\div$. It really consists of the process of finding how many times one number contains another.

As a simple example, divide 24 by 6.

Thus :—

$24 \div 6 = 4$ is really
$$\begin{aligned} 24 - 6 &= 18 \\ 18 - 6 &= 12 \\ 12 - 6 &= 6 \\ 6 - 6 &= 0 \end{aligned}$$
i.e., 6 could be successively subtracted 4 times from 24.

The number to be divided (i.e., 24 above) is called the **DIVIDEND**; the number by which it is to be divided (i.e., 6 above) is called the **DIVISOR,** and the result (i.e., 4 above) is called the **QUOTIENT.** Any quantity left over at the end of the operation is called the **REMAINDER.**

As a more difficult example, suppose we are required to divide 234 into 41184. We could say, by trial, that it would not go into 41184, two hundred times, but would go 100 times at least, with a quantity left over. By further trial, it would be seen that this quantity would be large enough for 234 to divide into it also, i.e., 234 would divide into 41184 more than 100 times. Let us set the operation down as far as we can go :—

```
234)41184(100
    23400
234)17784( 70
    16380
234) 1404(  6
     1404
     ----
```

5. **Division**—*continued.*

We can say that the number of times **234** divides into 41184, is 176. There is no need to set out the calculation this way, so long as we keep the place value of the figures in our mind. A more concise method of setting out is :—

```
     176
234)41184
    234
    ———
    1778
    1638
    ————
     1404
     1404
     ————
```

EXERCISES.

1. Write down, by inspection, the answers to the following divisions :—

(*a*) 1107 ÷ 9 ; (*b*) 6655 ÷ 11 ; (*c*) 2772 ÷ 12 ; (*d*) 429 ÷ 13 ; (*e*) 1512 ÷ 7.

2. Set out the following divisions in an orderly fashion :—

(*a*) 1073 ÷ 29 ; (*b*) 3224 ÷ 31 ; (*c*) 1848772 ÷ 7517.

3. In the following exercises, perform the division process, as for multiplication, before the addition and subtraction :—

(*a*) $60 - 4559 \div 47 + 37$; (*b*) $999 \div 37 - 15 + 43 \times 17 - 117$.

4. Suppose that in a certain city 2,470 applicants were successful for the Air Training Corps, and that it was decided to divide them up into 13 squadrons of equal numerical strength. How many would there be in each squadron ?

5. A report from America stated that the following rations had been ordered for 250,000 men to cover their period of training :—

30,000,000 eggs ; 7,000,000 lb. of meat ; 11,000,000 lb. of bread ; 9,500,000 lb. of potatoes ; 2,500,000 lb. of sugar, in addition to several other items.

How much of each item specified can be regarded as allowed per man ? The period of training was to cover 6 weeks. How much of each item can be considered as allowed each man per week ? Can you offer any comment on the fairness (or otherwise) of the share available to each man, as revealed by these figures ?

REVISION EXERCISES.

(These exercises are for further practice and revision of the processes studied in Lecture 1.)

1. Simplify :—

$763 - 8463 - 7128 - 317 + 10674 + 5832.$

2. The product of three numbers is 4301. Two of them are 11 and 17, find the third.

3. The population of a town at the beginning of a year was 596,238. There were 15,298 births and 13,569 deaths during the year. What was the population at the end of the year?

4. The following table gives the record of an army in a campaign :—

Number of men originally engaged	57,642
Number killed ..	3,261
Number wounded	8,634
Number missing ..	1,620

Determine the number still on active service.

5. Tribunals were set up throughout England to consider which aliens should be interned. During the first three months of this War, 40,000 cases were considered, but only 358 were interned. How many of those interviewed retained their freedom?

FUNDAMENTAL TABLES OF BRITISH UNITS.

1. TABLES.

A thorough knowledge of the units of measurement is essential. If any of the tables listed below are not known, they should be committed to memory. When wanted quickly, it will frequently be found impracticable to look them up.

Only the British units will be listed at this stage. The Metric (or Continental or Scientific) system will be given later in this preliminary course, in conjunction with the work on decimals.

Length.

12 inches	= 1	foot
3 feet	= 1	yard
22 yards	= 1	chain
10 chains	= 1	furlong
8 furlongs	= 1	mile
1760 yards, 5280 feet, 63360 inches	= 1	mile
6 feet	= 1	fathom
6080 feet	= 1	nautical mile

Time.

60 seconds	= 1	minute
60 minutes	= 1	hour
24 hours	= 1	day
7 days	= 1	week
52 weeks	= 1	year
365 days	= 1	year
366 days	= 1	leap year

Weight.

16 ounces	= 1	pound (lb.)
28 pounds	= 1	quarter (qr.)
4 quarters	= 1	hundred-weight (cwt.)
20 hundred-weight	= 1	ton
14 pounds	= 1	stone
112 pounds	= 1	hundred-weight
2240 pounds	= 1	ton

Square Measure.

144 sq. inches	= 1	sq. foot
9 sq. feet	= 1	sq. yard
4840 sq. yards	= 1	acre
640 acres	= 1	sq. mile

Cubic Measure.

1728 cub. inches	= 1	cub. foot
27 cub. feet	= 1	cub. yard

Circular Measure.

60 seconds	= 1	minute
60 minutes	= 1	degree
90 degrees	= 1	right angle
360 degrees	= 1	circle
	= 1	revolution

Measure of Capacity.

2 pints	= 1	quart
4 quarts	= 1	gallon
$6\frac{1}{4}$ gallons	= 1	cub. foot (Approx.)

Money.

12 pence	= 1	shilling
20 shillings	= 1	pound (£)
240 pence	= 1	pound.

2. USEFUL INFORMATION.

(1) One gallon of pure water weighs 10 lb.

(2) One cubic foot of water weighs $62\frac{1}{2}$ lb. approximately.

(3) One cubic foot of water corresponds to about $6\frac{1}{4}$ gallons.

(4) 6 feet = 1 fathom.

(5) 100 fathoms = 1 cable.

(6) 30 inches = 1 pace (military).

(7) The circumference of the earth at the equator = 24,899 miles (25,000 approximately).

(8) The diameter of the earth—

(*a*) measured through the poles = 7,900 miles approximately.

(*b*) measured at the equator = 7,927 miles approximately.

(9) A useful chart for remembering the number of days in the months of the year is as follows :—

31 days	Jan.	Mar.	May	July	Aug.	Oct.	Dec.
30 days		Apr.	June		Sept.	Nov.	
28 (or 29) days	Feb.						

(10) 60 nautical miles = 69·12 statute miles.

or $\dfrac{\text{Statute miles}}{66} = \dfrac{\text{Nautical miles}}{76}$ approximately.

EXAMPLES.

(1) Reduce 5 miles 4 furlongs to yards.

There are (5×1760) yards, i.e., 8800 yards in 5 miles, and (4×220) yards, i.e., 880 yards in 4 furlongs.

Hence there are $(8800 + 880)$ yards, i.e., 9680 yards in 5 miles 4 furlongs.

Or the calculation could also be performed :—

```
  5 miles 4 furlongs
  8             (8 furlongs = 1 mile.  Add the 4 furlongs)
 ----
 44 furlongs
 10             (10 chains = 1 furlong)
 ----
440 chains
 22             (22 yards = 1 chain)
 ----
880
880
 ----
9680 yards
```

Answer : 9680 yards.

Examples—*continued.*

(2) Convert 64,191 inches to miles, furlongs, chains, &c.

Set out as follows :—

12)64,191		(12 inches = 1 foot)
3) 5,349	+ 3 inches left over	(3 feet = 1 yard)
2) 1,783		
11) 891	+ 1 yard left over	(2 × 11 = 22 yards = 1 chain)
10) 81		(10 chains = 1 furlong)
8) 8	+ 1 chain left over	(8 furlongs = 1 mile)
1		

Answer.—1 mile 1 chain 1 yard 3 inches.

(3) Multiply 3 tons 2 cwt. 3 qrs. 5 lb. by 9.

Set out as follows :—

3 tons 2 cwt. 3 qrs. 5 lb.
9

28 tons 5 cwt. 0 qrs. 17 lb.

Each quantity is multiplied in turn by 9.
45 lb. = 1 quarter and 17 lb. over.
27 quarters + 1 quarter = 28 quarters = 7 cwt.
18 cwt. + 7 cwt. = 25 cwt. = 1 ton 5 cwt.
27 ton + 1 ton = 28 tons.

We could, if we wished, re-write our answer as—

28 tons 5 cwt. 1 stone 3 lb. (since 14 lb. = 1 stone).

EXERCISES.

1. Write down, by inspection, the answers to the following :—
   (*a*) the number of yards in 5 miles.
   (*b*) the number of miles in 126,720 inches.
   (*c*) the number of seconds in 1 hour.
   (*d*) the number of pounds in 10 tons.
   (*e*) the number of sq. yards in 108 sq. ft.
   (*f*) the number of degrees in three right angles.
   (*g*) the number of feet in 40 fathoms.
   (*h*) the weight of 10 gallons of water.
   (*i*) the number of days from 8th August to 31st October (1940—The Battle of Britain).
   (*j*) the number of feet in 2 nautical miles.

2. The stratosphere in the earth's atmosphere is said to begin at about 36,000 feet. How many miles is this ? Considering the number of feet left over, what is the answer to the **nearest** whole mile ?

3. Express a speed of 60 miles per hour in feet per second.

Exercises—*continued.*

4. A Heinkel He. III.K, Mark Va bomber is said to have an empty weight of 14,400 lb. How many tons, cwts., &c., does this weight represent? The wing area of the same aircraft is given as 942 sq. ft., how many sq. in. are there in this area?

5. On a well-known international map, one inch is taken to represent 1,000,000 inches on the ground. How many miles does this ground distance represent? From the number of inches left over, decide what the answer is to the nearest mile.

6. A trainee pilot on his first cross-country flight alone, was told to fly the three sides of a triangular course. Before setting out, he calculated that it would take him 35 minutes, 40 minutes, and 29 minutes to fly the three sides in order. If the complete journey actually took him 1 hour 50 minutes, how many minutes was he out in his reckoning?

7. A propeller rotated 1,800 times in a minute. How many times was this per second? How many degrees did it rotate through in 1 second?

8. It is recorded that in the early phases of the Battle of Britain, from 8th August to 5th September inclusive, no fewer than 4,523 aircraft were flown in daylight. How many aircraft does this represent on the average per day?

9. On an Air Force Station, 3,280 letters were posted at a cost of one-penny each. Express the money received in pounds, shillings and pence.

REVISION EXERCISES.

1. Add together:—
   £7 10s. $7\frac{1}{2}$d., £12 15s. 8d., £13 7s. 11d., £9 16s. $10\frac{1}{2}$d.

2. Express 378 quarters in tons, cwts. and quarters.

3. Multiply 6 miles 7 furlongs 176 yards by 10.

4. Mt. Everest is stated to be 29,000 feet high. How many feet is this less than six miles?

5. Goods have to be taken a distance of 50 miles. Of this, 9 miles 1,342 yards is by canal and 36 miles 570 yards by rail, and the remainder by cart. What distance is covered by cart?

MULTIPLES, FACTORS, FRACTIONS.

1. MULTIPLES AND FACTORS.

One number is said to be a **Multiple** of a second number when it contains the second number an **exact** number of times.

For example :—$24 = 6 \times 4 = 8 \times 3 = 12 \times 2 = 4 \times 3 \times 2$, &c.

In this case, 24 is a multiple of 6, or 4, or 8, or 3, &c. Similarly, 10 is a multiple of 5 and 2.

When several numbers multiplied together equal a given number, then these several numbers are said to be the **Factors** of the given number. Thus in the above example, 6 and 4 are factors of 24 Likewise, 8 and 3 or 12 and 2 or 4, 3 and 2 are also factors of 24

It should be noted that a number is always a multiple of its factors.

If a number has no factors except itself and unity, it is said to be a **PRIME NUMBER.**

Thus 17 is a prime number, as its only factors are 17 and 1. Similarly, 3, 5, 7, 11, &c., are also prime numbers.

When a number is expressed as the product of prime numbers, it is said to be resolved into its **PRIME FACTORS.** Thus in the above example, 24 expressed as prime factors, is :—

$$24 = 2 \times 2 \times 2 \times 3.$$

If a number is not a prime number, it can always be resolved into prime factors.

When a number is a multiple of two or more numbers, it is said to be the **COMMON MULTIPLE** of those numbers.

If it is the smallest number which is a common multiple of the other numbers, then it is called the **LOWEST COMMON MULTIPLE** (L.C.M.) of the other numbers.

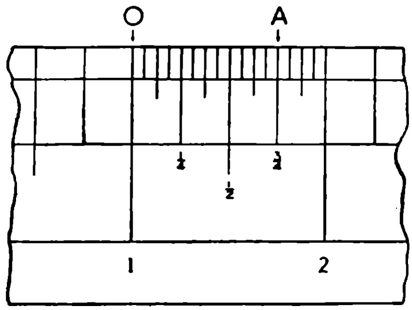
2. FRACTIONS.

If a quantity is divided into a number of equal parts, each part is said to be a **FRACTION** of the whole. Thus, if an inch is divided into ten equal parts, each division is said to be **one-tenth** (written $\frac{1}{10}$ or 1/10) of the inch.

Three of these divisions taken together would constitute three-tenths (written $\frac{3}{10}$ or 3/10) of the inch; five divisions, five-tenths; and so on.

2. **Fractions**—*continued.*

Examine the accompanying diagram, which represents an inch divided into sixteen equal parts (or sixteenths). The lengths of the division lines indicate other fractions into which the inch can be divided. Firstly, the inch is divided into two, giving halves (one-half $= \frac{1}{2}$); then into four, giving quarters (one-quarter $= \frac{1}{4}$); then into eight, giving eighths (one-eighth $= \frac{1}{8}$); and finally into sixteen, giving sixteenths (one-sixteenth $= \frac{1}{16}$).



The distance OA is three-quarters ($\frac{3}{4}$), six-eighths ($\frac{6}{8}$), or twelve-sixteenths ($\frac{12}{16}$) of an inch.

This division of the inch is taken by way of illustration. We can specify fractions of any quantity—$\frac{1}{2}$ ounce, $\frac{1}{3}$ mile, $\frac{1}{12}$ day, &c.; or, we can speak of the fraction of an object, $\frac{1}{4}$ apple, $\frac{3}{4}$ page, &c.

It can be seen that a fraction is expressed by two numbers:—

(A) the number of equal parts into which the given quantity is divided. This is called the **DENOMINATOR.** Thus 7 is the denominator in the fraction $\frac{1}{7}$.

(B) the number of the equal parts that are taken. This is called the **NUMERATOR.** Only one of the seven equal parts is taken in the fraction $\frac{1}{7}$, but three are taken in $\frac{3}{7}$.

Fractions of this kind are called **VULGAR FRACTIONS.** If the numerator is less than the denominator, then the fraction is called a **PROPER FRACTION** ($\frac{5}{6}$, $\frac{1}{3}$), while, if the numerator is greater than the denominator, it is called an **IMPROPER FRACTION** ($\frac{8}{7}$, $\frac{6}{5}$).

A **MIXED NUMBER** consists of a whole number and a proper fraction ($3\frac{1}{2}$, $2\frac{2}{3}$).

From the examination of the scale illustrated above, it can be seen that the fractions $\frac{3}{4}$, $\frac{6}{8}$, $\frac{12}{16}$, are the same. It should be noted that—

$$\frac{6}{8} = \frac{3 \times 2}{4 \times 2}$$

$$\frac{12}{16} = \frac{6 \times 2}{8 \times 2} = \frac{3 \times 4}{4 \times 4} = \frac{3 \times 2 \times 2}{4 \times 2 \times 2}$$

2. **Fractions**—*continued.*

Again, from a scale divided into tenths, you can confirm that $\frac{1}{2}$ and $\frac{5}{10}$ are the same and you can also see that $\frac{5}{10} = \frac{1 \times 5}{2 \times 5}$

From these examples, it will be seen that **the value of a fraction is unaltered by multiplying or dividing BOTH the numerator and the denominator by the same number.**

3. REDUCTION OF FRACTIONS.

Take a fraction such as $\frac{30}{42}$.

The prime factors of $30 = 2 \times 3 \times 5$, and of $42 = 2 \times 3 \times 7$. Hence the fraction may be rewritten :—

$$\frac{2 \times 3 \times 5}{2 \times 3 \times 7}$$

The 5 in the numerator and the 7 in the denominator are both multiplied by 2×3 and hence, from what was stated above, it can be seen that $\frac{5}{7}$ is the same as $\frac{30}{42}$. Thus $\frac{30}{42}$ is said to be reduced to its simplest form when it is written $\frac{5}{7}$. In practice, we usually bring about this reduction by inspection, thus :—

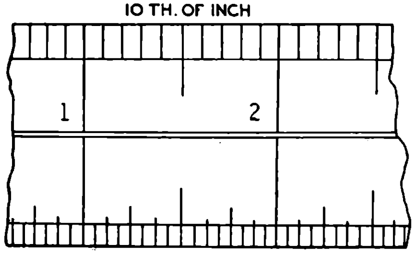
$$\frac{30}{42} = \frac{15}{21} = \frac{5}{7} \text{ or } \frac{\overset{5}{\overset{\cancel{15}}{\cancel{30}}}}{\underset{7}{\underset{\cancel{21}}{\cancel{42}}}} = \frac{5}{7}.$$

We can say that we have **cancelled** out, first the common factor 2, then the common factor 3, thus leaving the numerator and the denominator without any common factors. When a fraction is in this form, we say that it has been **reduced to its lowest terms.**

4. ADDITION AND SUBTRACTION OF FRACTIONS.

The accompanying diagram represents a scale whose upper edge is divided into tenths of inches and the lower into sixteenths. It can readily be seen that $\frac{1}{2} + \frac{1}{8}$ is certainly **not** $\frac{1}{10}$. How are we then to add these two fractions to give the correct answer, which, from the scale, can be seen to be $\frac{5}{8}$? From what has been done above,

10 TH. OF INCH



16TH OF INCH

4. Addition and Subtraction of Fractions—*continued.*

we can see that $\frac{1}{2}$ is the same as $\frac{4}{8}$ and hence, if $\frac{1}{2}$ is considered as four-eighths ($\frac{4}{8}$) and we add the one-eighth ($\frac{1}{8}$), then we will obtain ($4 + 1 = 5$) five-eighths. Both fractions were expressed with **the same denominator.** Take another example :—

$$\frac{1}{5} + \frac{3}{10}$$
$$\frac{1}{5} = \frac{2}{10}$$
$$\text{Hence } \frac{1}{5} + \frac{3}{10} = \frac{2}{10} + \frac{3}{10} = \frac{5}{10}.$$

As we can see from the scale, and also by cancelling, $\frac{5}{10} = \frac{1}{2}$. Again the fractions were converted to **the same denominator** to effect the addition.

Taking another example, it is evident that $\frac{1}{7} + \frac{1}{4}$ cannot be simplified until we write both the fractions with **a common denominator,** which is a number that contains each of the denominators of the given fractions an exact number of times. The common denominator need not be the same as one of the denominators itself, as it was in the examples above. In this case it is 28.

$$\frac{1}{7} = \frac{4}{28},\ \frac{1}{4} = \frac{7}{28}.$$
$$\text{Hence } \frac{1}{7} + \frac{1}{4} = \frac{4}{28} + \frac{7}{28} = \frac{11}{28}.$$

In order to simplify work, it is desirable to keep the common denominator as low as possible. For this reason we seek the **LEAST COMMON DENOMINATOR** (L.C.D.) or **LEAST COMMON MULTIPLE** (L.C.M.) of the denominator.

Take as an example, $\frac{3}{4} + \frac{5}{6} + \frac{7}{8}$.

By inspection we can see that 192 (i.e., $4 \times 6 \times 8$) is a multiple of 4, 6 and 8 and could be taken as a common denominator, but it is not **the least.** Likewise, 96 and 48 can be divided equally by all the denominators, 4, 6 and 8, but they too are not the least. Eventually it can be seen that 24 is the **least** number into which 4, 6 and 8 will all divide, i.e., 24 is the required **least common denominator.**

The numbers here are easy. The L.C.D. cannot always be determined simply by inspection. Consequently, it will be as well to set out a method for finding the L.C.D. that can be employed with any numbers.

The numbers in the denominators are expressed in terms of their prime factors :—

$4 = 2 \times 2 \quad = 2^2$ (2^2 is written thus for conciseness.
$6 = 2 \times 3 \quad = 2 \times 3$ See the Algebra course for further
$8 = 2 \times 2 \times 2 = 2^3$ reference to this fact.)

Taking the separate factors of each number in turn, it can be seen that 2^3 is the only one into which each of the powers of 2 will divide exactly, so 2^3 must be included in the L.C.D.

There is only one other factor (3) and this will likewise have to be included in the L.C.D.

4. Addition and Subtraction of Fractions—*continued.*

The L.C.D. $= 2^3 \times 3 = 8 \times 3 = 24.$

$\frac{3}{4} + \frac{5}{6} + \frac{7}{8} = \frac{18}{24} + \frac{20}{24} + \frac{21}{24}$ (or $\dfrac{18 + 20 + 21}{24}$ for conciseness)

$= \frac{59}{24}$ (an improper fraction) $= 2\frac{11}{24}$ (mixed number).

Take another example, involving both addition and subtraction of fractions—

$\frac{1}{2} + \frac{3}{4} - \frac{5}{6} + \frac{3}{8} - \frac{7}{15}.$

Finding the L.C.D.

Since 2 and 4 divide into 8 it is enough for us to find the L.C.D. of 6, 8 and 15—

$$6 = 2 \times 3$$
$$8 = 2^3$$
$$15 = \quad 3 \times 5.$$

Therefore L.C.D. $= 2^3 \times 3 \times 5$
$= 8 \times 3 \times 5$
$= 120.$

$$\text{Then, } \frac{1}{2} + \frac{3}{4} - \frac{5}{6} + \frac{3}{8} - \frac{7}{15} = \frac{60 + 90 - 100 + 45 - 56}{120}$$
$$= \frac{195 - 156}{120}$$
$$= \frac{39}{120}$$

NOTE.—Always reduce your answer to its lowest terms.

Thus in above example—

3 divides into both 39 and 120, giving $\frac{39}{120} = \frac{13}{40}$.

Answer $= \frac{13}{40}$.

To compare the sizes of fractions, we can convert them to a common denominator. They can then be arranged according to the size of their numerators.

Example.—Which is the greatest fraction—$\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$.

By inspection, or by the factor method, **L.C.D.** $= 60$.

$\frac{3}{4} = \frac{45}{60}$, $\frac{4}{5} = \frac{48}{60}$, $\frac{5}{6} = \frac{50}{60}$.

Consequently, it can be seen $\frac{5}{6}$ is the greatest fraction.

EXERCISES.

1. The answers to the following exercises should be written down by inspection :—

(*a*) Express in their simplest forms—$\frac{6}{9}$, $\frac{8}{10}$, $\frac{8}{64}$, $\frac{5}{5}$, $\frac{27}{33}$.

(*b*) Express as mixed numbers—$\frac{4}{3}$, $\frac{6}{5}$, $\frac{13}{4}$, $\frac{11}{7}$, $\frac{76}{11}$.

(*c*) Express as improper fractions—$1\frac{1}{3}$, $2\frac{1}{2}$, $1\frac{3}{5}$, $12\frac{1}{12}$, $9\frac{7}{9}$.

(*d*) Express as fractions with the same denominator—$\frac{1}{2}$ and $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{1}{3}$, $\frac{1}{5}$ and $\frac{1}{2}$, $\frac{11}{12}$ and $\frac{7}{6}$, $\frac{1}{2}$ and $\frac{1}{4}$ and $\frac{1}{8}$.

(*e*) $\frac{1}{4} + \frac{1}{3} =$, $\frac{1}{2} + \frac{1}{6} + \frac{1}{3} =$, $\frac{5}{6} - \frac{1}{3} =$

Exercises—*continued.*

2. Simplify the following :—

(*a*) $\frac{3}{5} + \frac{1}{25} + \frac{7}{75}$.
(*b*) $\frac{3}{7} + \frac{5}{9} + \frac{5}{8} + \frac{5}{72}$.
(*c*) $\frac{2}{5} + \frac{7}{12} + \frac{9}{10} + \frac{13}{20}$.
(*d*) $\frac{3}{8} + \frac{5}{9} - \frac{7}{24}$.
(*e*) $\frac{5}{121} - \frac{1}{33}$.
(*f*) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{5}$.

(**NOTE.**—In adding and subtracting mixed numbers, it is usually easier and may save a lot of unnecessary work to consider the whole numbers separately—thus $3\frac{1}{6} + 1\frac{3}{4} - 2\frac{1}{3} = 3 + 1 - 2 + \frac{1}{6} + \frac{3}{4} - \frac{1}{3}$.)

(*g*) $3\frac{1}{3} + 4\frac{1}{9} - 6\frac{1}{6}$.
(*h*) $3\frac{1}{12} + 5\frac{3}{8} + \frac{3}{16}$.
(*i*) $\frac{27}{2} - \frac{16}{3} + \frac{28}{6}$.
(*j*) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$.

REVISION EXERCISES.

1. Arrange the following fractions in order, giving the largest first :—

$\frac{5}{7}$, $\frac{9}{14}$, $\frac{13}{16}$.

2. Find the prime factors of 504 and of 4095.

3. Subtract $\frac{5}{8}$ from the sum of $\frac{1}{2} + \frac{1}{4} + \frac{3}{8}$.

4. A wooden pile was driven so that $\frac{2}{9}$ of its length was in the ground, and $\frac{3}{7}$ in water. What fraction of its length protruded above the water ?

5. Subtract the sum of $\frac{4}{5}$ and $7\frac{5}{8}$ from $17\frac{2}{3}$.

MULTIPLICATION AND DIVISION OF FRACTIONS.

1. To **multiply a fraction by a whole number,** multiply the numerator by the whole number.

Thus, $\frac{3}{8} \times 3 = \frac{9}{8} = 1\frac{1}{8}$.

We can see this, if we express the operation as $\frac{3}{8}$ repeated 3 times, or $\frac{3}{8} + \frac{3}{8} + \frac{3}{8}$. You can check the result $\frac{9}{8}$ from the scales illustrated in Lecture 3 by noting the range of 3 consecutive $\frac{3}{8}$ intervals.

2. To **multiply one fraction by another fraction,** we multiply the given numerators to find the numerator of the product, and we multiply the given denominators to find the new denominator.

Thus, $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

Note.—The word "**of**" is sometimes used instead of the sign $\times$. Thus, $\frac{2}{3}$ of $\frac{4}{5}$ has the same value as $\frac{2}{3} \times \frac{4}{5}$, but further mention will be made later as to the use of terms joined by "of" in conjunction with other terms.

Often tedious multiplications can be avoided if we inspect the fractions given, to observe whether cancelling can be performed on them.

$$\text{Thus, } \frac{12}{25} \times \frac{35}{44} = \frac{3 \times 4}{5 \times 5} \times \frac{5 \times 7}{4 \times 11} = \frac{3 \times \cancel{4} \times \cancel{5} \times 7}{\cancel{5} \times 5 \times \cancel{4} \times 11} = \frac{21}{55}$$

More compactly—

$$\frac{\overset{3}{\cancel{12}}}{\underset{5}{\cancel{25}}} \times \frac{\overset{7}{\cancel{35}}}{\underset{11}{\cancel{44}}} = \frac{21}{55}$$

It is in the last method particularly that attention should be given to neatness in arranging the figures, otherwise errors easily creep in.

3. To **divide a fraction by a whole number,** multiply the denominator by the number—

$$\text{thus } \frac{3}{8} \div 2 = \frac{3}{8} \times \frac{1}{2} = \frac{3}{8 \times 2} = \frac{3}{16}$$

Referring to the scale diagrams in Lecture 3, we can see that if we take any interval of three one-eighth divisions and divide it into two equal parts, each part contains three-sixteenths. Another way of expressing the process is to say that we multiply the given fraction by the **reciprocal** of the number.

When the product of two numbers (whole or fractional) is 1, each is called the reciprocal of the other.

Thus, $\frac{1}{2}$ is the reciprocal of 2, because $\frac{1}{2} \times 2 = 1$.

Likewise, $\frac{3}{5}$ and $\frac{5}{3}$ are reciprocals, because $\frac{3}{5} \times \frac{5}{3} = 1$.

4. To **divide a fraction by another fraction,** multiply the one by the reciprocal of the other.

$$\text{Thus, } \frac{5}{9} \div \frac{4}{7} = \frac{5}{9} \times \frac{7}{4} = \frac{35}{36}$$

Multiplication and Division of Fractions—*continued.*

5. **Mixed numbers** should be converted into improper fractions before multiplying or dividing.

$$\text{Thus, } 3\tfrac{1}{7} \div 3\tfrac{2}{3} = \tfrac{22}{7} \div \tfrac{11}{3} = \frac{\overset{2}{\cancel{22}}}{7} \times \frac{3}{\underset{1}{\cancel{11}}} = \tfrac{6}{7}.$$

6. As was expressed for whole numbers in Lecture 1, so for fractions we must :—

(1) Perform **multiplications and divisions** before additions and subtractions **when they occur together.**

e.g., **simplify**: $\frac{3}{7} + \frac{5}{6} \times \frac{14}{15}$

$$\frac{3}{7} + \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}} \times \frac{\overset{7}{\cancel{14}}}{\underset{3}{\cancel{15}}} = \frac{3}{7} + \frac{7}{9} = \frac{27 + 49}{63} = \frac{76}{63} = 1\tfrac{13}{63}.$$

(2) Perform operations inside brackets **first.** Brackets indicate that the expression enclosed within it is to be treated as a single quantity.

e.g., simplify: $(3\frac{1}{4} + 2\frac{1}{2}) \div 5\frac{1}{2}$

$$\begin{aligned}(3\tfrac{1}{4} + 2\tfrac{1}{2}) \div 5\tfrac{1}{2} &= 5\tfrac{3}{4} \div 5\tfrac{1}{2}\\ &= \tfrac{23}{4} \div \tfrac{11}{2}\\ &= \frac{23}{\underset{2}{\cancel{4}}} \times \frac{\cancel{2}}{11}\\ &= \tfrac{23}{22}\\ &= 1\tfrac{1}{22}.\end{aligned}$$

Various types of brackets are used, but they are treated fully in the Algebra section of this course.

(3) Regard terms joined by "of" as though they were in brackets and to be multiplied. Consequently, they must be worked first in a problem containing other operations.

A useful comparison may be made between the following two problems :—

(*a*) $\frac{1}{3} \times \frac{2}{3} \div \frac{4}{5}$ of $\frac{3}{2}$

$$\begin{aligned}&= \tfrac{1}{3} \times \tfrac{2}{3} \div \left(\frac{\overset{2}{\cancel{4}}}{5} \times \frac{3}{\underset{1}{\cancel{2}}}\right)\\ &= \tfrac{2}{9} \div \tfrac{6}{5}\\ &= \frac{\overset{1}{\cancel{2}}}{9} \times \frac{5}{\underset{3}{\cancel{6}}}\\ &= \tfrac{5}{27}\end{aligned}$$

(*b*) $\frac{1}{3} \times \frac{2}{3} \div \frac{4}{5} \times \frac{3}{2}$

$$\begin{aligned}&= \frac{1}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{2}}}{3} \times \frac{5}{4} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}}\\ &= \tfrac{5}{12}\end{aligned}$$

In example (*a*), since $\frac{4}{5}$ and $\frac{3}{2}$ are joined by "of," their product must be regarded as one term for the purpose of the division. For this reason the operation $\frac{4}{5} \times \frac{3}{2}$ was performed within brackets.

Multiplication and Division of Fractions—*continued.*

In example (*b*), the multiplication and division are taken in the order in which they occur. The division sign applies only to $\frac{4}{5}$ and so only the $\frac{4}{5}$ is inverted before replacing the $\div$ sign by the $\times$ sign.

7. **Summarizing** then the rules for **the order of working** in problems involving a mixture of brackets, the term "of," and the signs, $\times$, $\div$, $+$, and $-$, we have :—

(i) Simplify inside all **brackets.**

(ii) Replace the "**ofs**" by multiplication signs and treat the fractions joined by them as if they were bracketed.

(iii) Take the $\times$ and $\div$ operations as they occur.

(iv) Perform the operations for $+$ and $-$ signs.

EXAMPLE.

Simplify: $3\frac{3}{7} \div 5\frac{5}{7}$ of $\frac{7}{32} - 2\frac{2}{7}$

$$3\frac{3}{7} \div 5\frac{5}{7} \text{ of } \frac{7}{32} - 2\frac{2}{7} = \frac{24}{7} \div \left(\frac{\overset{5}{\cancel{40}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{4}{\cancel{32}}}\right) - \frac{16}{7}$$

$$= \frac{24}{7} \div \frac{5}{4} - \frac{16}{7}$$

$$= \frac{24}{7} \times \frac{4}{5} - \frac{16}{7}$$

$$= \frac{96}{35} - \frac{16}{7}$$

$$= \frac{96 - 80}{35}$$

$$= \frac{16}{35}.$$

EXERCISES.

1. Write down, by inspection, the answers to the following :—

(*a*) $\frac{9}{10}$ of £1, $\frac{3}{8}$ of 1 lb., $\frac{5}{9}$ of 1 yd., $\frac{1}{3}$ of 1 sq. yd., $\frac{3}{4}$ of one complete revolution (in degrees).

(*b*) Multiply: $\frac{1}{3} \times \frac{2}{3}$, $\frac{1}{3} \times \frac{3}{5}$, $\frac{1}{4} \times \frac{2}{5}$, $\frac{3}{8} \times \frac{7}{9}$, $1\frac{1}{2} \times \frac{2}{3}$.

(*c*) Divide: $\frac{1}{2}$ by 3, $\frac{3}{4}$ by 3, 1 by $\frac{2}{5}$, $\frac{5}{8}$ by $\frac{5}{4}$, $\frac{6}{7}$ by $\frac{7}{6}$.

(*d*) Give the reciprocals of: 3, $\frac{1}{6}$, $\frac{2}{7}$, $\frac{5}{9}$, $\frac{7}{8}$.

(*e*) Simplify: $5\frac{1}{2} - (3\frac{1}{4} + 2)$.

2. Simplify :—

(*a*) $\frac{25}{26} \times \frac{27}{35}$, $5\frac{2}{3} \times \frac{11}{51}$.

(*b*) $\frac{5}{9} \div \frac{7}{8}$, $1\frac{4}{5} \div 4\frac{1}{2}$.

(*c*) $3\frac{1}{5}$ of $4\frac{2}{7}$.

(*d*) $\dfrac{3 + 1\frac{1}{3}}{3 - 1\frac{1}{3}}$

(*e*) $(6\frac{1}{6} - 4\frac{3}{4}) \div 2\frac{1}{3}$.

(*f*) $3\frac{2}{3}$ of $5\frac{1}{9} \times \frac{27}{46}$.

(*g*) $2\frac{1}{4}$ of $1\frac{1}{2} \div 1\frac{1}{3}$ of $2\frac{2}{5}$.

(*h*) $(\frac{3}{4} + \frac{1}{2}) \div \frac{3}{8}$ of $\frac{1}{6}$.

(*i*) $\frac{1}{15}$ of $3\frac{1}{3} + \frac{1}{3}$.

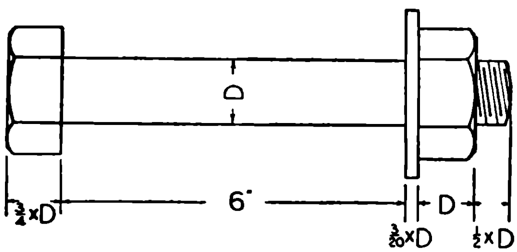
(*j*) $5\frac{1}{2} + 3\frac{1}{3} \times 4\frac{1}{5} - 7$ of $1\frac{1}{14}$.

REVISION EXERCISES.

1. On 15th September, 1940, it was estimated that 500 German aircraft engaged in a running fight with English Hurricanes and Spitfires. It is known that the enemy losses were at least 185 aircraft. Express this loss as a fraction of the German aircraft that came over. Reduce the fraction. What fraction to our knowledge escaped ?

2. The crew of the auxiliary vessel *Northern Rover*, which was reported lost early in this war, comprised a Captain, three other officers and twenty-three ratings. What fraction of the crew was officers ?

3. The accompanying drawing is of a bolt. The dimensions of the various parts are given in terms of D (the diameter of the bolt stem). If D = $1\frac{1}{2}$ inches, what is the over-all length of the bolt ?



4. The scale on a map is 1 inch = 4 miles. How many miles would be represented by $\frac{7}{10}$ inch on the map ? Convert the fraction of a mile in the answer to yards.

5. In London over a certain period $\frac{1}{25}$ of the men applying for exemption from military service on certain grounds were successful in their applications. If there were 32 successful, how many men originally applied for this exemption ?

DECIMALS.

1. In Lecture 1 we saw that, for every place we shifted a figure **to the left** of the units position in a number, we **increased** its value ten times.

Conversely, for every place we shift a figure **to the right** in a number we **decrease** its value ten times.

Thus in the number 7265, the figure 2 represents 200, i.e., 2×100. In 7526, it represents 20, or 2×10, whilst in 7562 it represents only 2×1.

If we were again to **decrease** its value ten times, its value, as we now know, would be two-tenths, which could be represented by $\frac{2}{10}$. However, we can extend the idea of place value and simply shift it one place to the right of the units position. A convention of placing a dot between the units position and the tenths position is employed. Thus in the number 1756·2, the figure 2 now represents two-tenths.

For every further place we shift it to the right we **decrease** its value a further ten times. In the number 75·62 it represents two-hundredths (i.e., $\frac{2}{100}$).

The dot, or point, is called a **decimal point.** Thus we can read 75·62 to mean "seventy-five point six-two".

A number such as 75·62 is a **mixed number** and the portion following the units place is a **decimal fraction** or decimal.

The number $75{\cdot}62 = 70 + 5 + \frac{6}{10} + \frac{2}{100}$.

But, $\frac{6}{10} + \frac{2}{100} = \dfrac{6 \times 10}{100} + \frac{2}{100} = \frac{60}{100} + \frac{2}{100} = \frac{62}{100}$, so that we could convert 75·62 to $75\frac{62}{100}$ if we wished.

In a similar manner it may be shown that

$$1756{\cdot}2 = 1756\tfrac{2}{10}$$
$$2{\cdot}123 = 2\tfrac{123}{1000}$$
$$\cdot 02 = \tfrac{2}{100},$$

and so on.

2. ADDITION AND SUBTRACTION OF DECIMALS.

In adding or subtracting numbers containing decimals, we use precisely the same method as given in Lecture 1, remembering now to keep the tenths one under the other, hundredths below hundredths and so on.

e.g., adding 14·632, 271·9 and 388·09 we have—

```
 14·632
271·9
388·09
-------
674·622
-------
```

and subtracting 271·097 from 338·906 we have—

```
338·906
271·097
-------
 67·809
-------
```

3. MULTIPLICATION OF DECIMALS.

The multiplication of decimal quantities is carried out as for whole numbers, the decimal point being fixed in the answer by remembering that there are as many decimal places in the answer as there are in the multiplier and multiplicand together.

The decimal point in the answer will fall vertically below the decimal point in the multiplicand if the figure in the units place in the multiplier is always placed under the last figure on the right in the multiplicand.

e.g., (*a*) $36{\cdot}73 \times 4{\cdot}509$

$$\begin{array}{l} 36{\cdot}73 \\ 4{\cdot}509 \\ \hline 146{\cdot}92 \\ 18{\cdot}365 \\ {\cdot}33057 \\ \hline 165{\cdot}61557 \\ \hline \end{array}$$

(*b*) $27{\cdot}98 \times 63{\cdot}5$

$$\begin{array}{r} 27{\cdot}98 \\ 63{\cdot}5 \\ \hline 1678{\cdot}8 \\ 83{\cdot}94 \\ 13{\cdot}990 \\ \hline 1776{\cdot}730 \\ \hline \end{array}$$

Notice.—We are still multiplying "from the left."

Sometimes noughts (or ciphers) have to be placed to the left of a decimal answer to make up the correct number of decimal places.

e.g., $\cdot 36 \times \cdot 045$—

$$\begin{array}{l} {\cdot}36 \\ {\cdot}045 \\ \hline {\cdot}0144 \\ {\cdot}00180 \\ \hline {\cdot}01620 \\ \hline \end{array}$$

Notice the position of the multiplier.

Why is it so placed ?

4. STANDARD FORMS.

(*a*) A decimal quantity is said to have been standardized (or written in a standard form) when it is so rearranged that it has only a units digit to the left of the decimal point, and the rearranged number is multiplied or divided by 10 or powers of 10 according as the decimal point is moved to the left or right.

Thus $26{\cdot}08$ in the standard form is $2{\cdot}608 \times 10$. By shifting each of the figures one place to **the right** with reference to the decimal point we have **decreased** their values ten times. Consequently to maintain the original value we must **multiply** the new value by ten.

If the given number had been $\cdot 2608$, the standard form would have become $2{\cdot}608 \div 10$.

EXAMPLES.

260·8 in the **standard** form is $2{\cdot}608 \times 100$, while ·02608 is $2{\cdot}608 \div 100$.

Likewise, the standard **forms of** ·0021 and 2100 are $2{\cdot}1 \div 1000$ and $2{\cdot}100 \times 1000$ respectively.

Note.—Noughts to the extreme **right of a** decimal fraction do not alter its actual numerical value, thus **2·1, 2·10, 2·100 are** all the same. But the figure 1 in numbers like 2·01, 2·001 **has its** place value altered by the insertion of noughts in this way.

(*b*) We can write $100 = 10 \times 10 = 10^2$,
$1000 = 10 \times 10 \times 10 = 10^3$, &c.

This concise notation of 10^2 for 100, 10^3 for 1000, &c., **will be frequently** employed in future, but before proceeding, notice **that** $100 \times 1000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$, so that we say $10^2 \times 10^3 = 10^5$. The numbers 2 and 3 are each called **an index or power (plural, indices).**

Hence to **multiply** powers of 10 **we need only express** them in their index forms and **add** the indices.

$10 = 10^1$, hence $10 \times 1000 = 10^1 \times 10^3 = 10^4$ (or 10,000).

We **subtract** the indices for **division.**

$$(10^4) \div (10^2) = \frac{10{,}000}{100} = \frac{\cancel{10} \times \cancel{10} \times 10 \times 10}{\cancel{10} \times \cancel{10}} = 10 \times 10 = 10^2.$$

$$\frac{(10^2)}{(10^4)} = \frac{\cancel{10} \times \cancel{10}}{\cancel{10} \times \cancel{10} \times 10 \times 10} = \frac{1}{10 \times 10} = \frac{1}{10^2}$$

Now applying standard forms to calculations involving **decimals,** we have—

$$\begin{aligned} &2{,}670{,}000 \times {\cdot}00032 \\ &= (2{\cdot}67 \times 10^6) \times (3{\cdot}2 \div 10^4) \\ &= 2{\cdot}67 \times 3{\cdot}2 \times 10^6 \div 10^4 \\ &= 2{\cdot}67 \times 3{\cdot}2 \times 10^2 \\ &= 8{\cdot}544 \times 10^2 \\ &= 854{\cdot}4 \end{aligned}$$

$$\begin{array}{r} 2{\cdot}67 \\ 3{\cdot}2 \\ \hline 8{\cdot}01 \\ {\cdot}534 \\ \hline 8{\cdot}544 \\ \hline \end{array}$$

or again—

$$\begin{aligned} &{\cdot}036 \times {\cdot}0098 \\ &= (3{\cdot}6 \div 10^2) \times (9{\cdot}8 \div 10^3) \\ &= 3{\cdot}6 \times 9{\cdot}8 \div 10^2 \div 10^3 \\ &= 35{\cdot}28 \div 10^5 \\ &= {\cdot}0003528 \end{aligned}$$

$$\begin{array}{r} 3{\cdot}6 \\ 9{\cdot}8 \\ \hline 32{\cdot}4 \\ 2{\cdot}88 \\ \hline 35{\cdot}28 \\ \hline \end{array}$$

5. DIVISION OF DECIMALS.

The division is carried out as for whole numbers, but the decimal place in the answer is fixed by shifting the decimal point in both the divisor and dividend the same number of places to the right, in order to convert the divisor to a whole number.

EXAMPLES.

(1) $14{\cdot}825 \div 59{\cdot}3$

$148{\cdot}25 \div 593$

```
        ·25
593)148·25
     118·6
     -----
      29·65
      29·65
      -----
```

Answer: ·25

(2) $87{\cdot}6618 \div 3{\cdot}142$

$87661{\cdot}8 \div 3142$

```
          27·9
3142)87661·8
      6284
      -----
      24821
      21994
      -----
       28278
       28278
       -----
```

Answer: 27·9

(3) $0{\cdot}876618 \div 0{\cdot}003142$

$876618 \div 3142$

```
          279
3142)876618
      6284
      -----
      24821
      21994
      -----
       28278
       28278
       -----
```

Answer: 279

Division of decimals may also be worked in the standard form.

Take as an example, $3670{\cdot}92 \div 89{\cdot}1$:

$$\frac{3{\cdot}67092 \times 10^3}{8{\cdot}91 \times 10} = \frac{3{\cdot}67092 \times 10^2}{8{\cdot}91}$$

```
       ·412
8·91)3·67092
     3·564
     -----
      ·1069
       891
      -----
        1782
        1782
       -----
```

Answer: $\cdot 412 \times 10^2 = \mathbf{41{\cdot}2}$

6. EXPRESSION OF A DECIMAL FRACTION AS A VULGAR FRACTION.

e.g., express $\cdot 125$ as a vulgar fraction—

$$\cdot 125 = \tfrac{125}{1000} = \tfrac{1}{8}.$$

7. EXPRESSION OF A VULGAR FRACTION AS A DECIMAL FRACTION.

Divide the numerator by the denominator.

e.g., express $\frac{3}{5}$ as a decimal fraction—

$$5)\underline{3\cdot 0} \qquad \tfrac{3}{5} = \cdot 6$$
$$\cdot 6$$

EXERCISES.

1. Write down, by inspection, the answers to the following :—
   (*a*) $\cdot 3 \times 21$; (*b*) $\cdot 3 \times \cdot 2$; (*c*) $\cdot 05 \times \cdot 3$; (*d*) $\cdot 02 \times \cdot 03$; (*e*) $\cdot 325 + \cdot 29$; (*f*) $\cdot 34 - \cdot 28$; (*g*) $\cdot 8 \div 2$; (*h*) $\cdot 8 \div \cdot 2$; (*i*) $\cdot 08 \div \cdot 2$; (*j*) $50 \div \cdot 25$.

2. Express the following decimal quantities as vulgar fractions :—
   $\cdot 25$; $\cdot 5$; $\cdot 75$; $12\cdot 17$.

3. Express the following vulgar fractions as decimal fractions :—
   $\frac{3}{8}$, $\frac{2}{5}$, $\frac{5}{8}$, $\frac{1}{16}$, $\frac{9}{10}$.

4. Express the following in standard form :—
   $39\cdot 32$; $393\cdot 2$; $\cdot 3932$; 103,932; 3,932,000.

5. Calculate the following :—
   (*a*) $3\cdot 76 \times 8\cdot 23$;
   (*b*) $31\cdot 85 \times 9\cdot 762$;
   (*c*) $216\cdot 071 \times 3\cdot 142$;
   (*d*) $256\cdot 732 \times \cdot 7854$;
   (*e*) $\cdot 034 \times \cdot 237$;
   (*f*) $18\cdot 096 \div \cdot 78$;
   (*g*) $\cdot 00945 \div \cdot 045$;
   (*h*) $2198\cdot 942 \div 3\cdot 14$;
   (*i*) $\cdot 0391 \div \cdot 0023$;
   (*j*) $\dfrac{6\cdot 24 \times 13\cdot 34}{\cdot 52}$;
   (*k*) $\cdot 35 + \cdot 525 - \cdot 75$;
   (*l*) $1\cdot 732 - 1\cdot 414$.

REVISION EXERCISES.

1. Express $3\cdot 725$ as an improper fraction.

2. Express $\cdot 625$ as a vulgar fraction.

3. Write down the values of the following :—
   $\cdot 25$ of £1; $\cdot 2$ of 1 ton, in cwts.; $\cdot 5$ of $1\frac{1}{2}$ hours, in minutes $\cdot 1$ of 1 mile, in yards; $\cdot 125$ of 1 cwt. in lbs.

4. $591\cdot 23 \times \cdot 752$; $1\cdot 414 \times 1\cdot 732$; $\cdot 057 \times \cdot 239$.

5. $\dfrac{\cdot 01488 \times 70\cdot 2}{\cdot 048}$.

Revision Exercises—*continued.*

6. An aircraft engine, with an ordinary intake of air, is said to have at 20,000 ft. about ·47 of the horse-power developed at sea-level. If a certain engine had a horse-power of 800 at sea-level, what would be the value at 20,000 ft.?

7. The following official records were made of observations on the speeds achieved by G. H. Stainforth in the R.A.F. Vickers-Supermarine when he broke, in 1931, the world's speed record in the air:—415·2, 407·4, 409·5, 407·6 and 404·5 m.p.h. What average speed does the last four readings indicate? The first one had to be discarded, but if it had been retained, what average would the first four readings including it have given? Give each of the answers to the first decimal place.

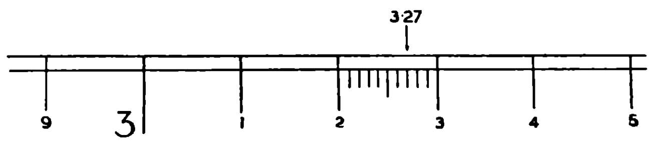
APPROXIMATIONS—SIGNIFICANT FIGURES.

1. APPROXIMATIONS.

It is often desirable to express a number in an approximate form. The extent to which the approximation is carried depends on circumstances. Thus, in one problem we may desire to state a calculated length, say, 3·27 inches, to the nearest inch, and in another, to the nearest first decimal place.

3·27 inches is a length that is greater than 3 inches by ·27 inch but less than 4 inches by ·73 inch. Consequently, to state its value to the **nearest** inch we should give it as 3 inches.

The truth of these statements may be observed in the illustration below, which is purely diagrammatic. In actual measurement, with a scale marked in tenths of an inch, we must **estimate** the hundredths but, by means of a vernier, we can **measure** to a hundredth of an inch.



That 3·27 is 3·3, correct to one decimal place, could have been written down by inspection. The method for doing this is to look at the second decimal place and if it has a value **5 or over,** simply **add 1** to the first decimal place. If the value of the second decimal place were **less than 5** (say, 3·23), **make no alteration** to the first decimal place. The answer in this latter case, to the first decimal place, would be **3·2.**

EXAMPLES.

153·2 correct to the nearest whole unit is 153.
153·7 correct to the nearest whole unit is 154.
3·14159 correct to the third decimal place is 3·142.
·0362 correct to the second decimal place is ·04.
92,965,000 correct to the nearest million is 93,000,000.

2. ESTIMATED ANSWERS.

It is most desirable to form the habit of making an estimate of the answer in any calculation before actually working it. Besides giving a useful guide as to whether the result itself is reasonably correct, it helps in fixing the decimal point in the answer.

e.g., (*a*) estimate the answer for 152·7 $\div$ 3·142.

152·7 is slightly greater than 150 and 3·142 is slightly greater than 3.

$\dfrac{152{\cdot}7}{3{\cdot}142}$ is therefore approximately equal to $\dfrac{150}{3}$,

or **50.** (Actually the answer to the calculation is **48·6.**)

2. **Estimated Answers**—*continued.*

(*b*) 17·25 × ·792.

17·25 is a little less than 20, and ·792 is almost equal to ·8.

Hence, 17·25 × ·792 is approximately equal to 20 × ·8, or **16.** (The actual answer is **13·662.**)

3. **SIGNIFICANT FIGURES.**

It was stated in Paragraph 1 that, to obtain the value 3·27 inches from a scale graduated in tenths of an inch only, we should have to **estimate** the hundredths by eye. Since the 7 is only an estimate, its value would not be as reliable as the 2 given in the tenths position, the value of which we can actually read off from the division marks on the scale. We depend, of course, for our accuracy of the measurement upon the accuracy of construction of the scale itself. The smaller the subdivisions on a scale, the more we depend upon the precision of workmanship. In general, we might say the accuracy of any measurement is limited by the accuracy of the instrument used.

For this reason, in giving an answer to a calculation involving measurement, we should consider what figures have **significance,** or indicate actual measurement. In our example above, the 2 has more significance than the 7.

In any measurement, the first figure on the left has the most significance and is termed the first **significant figure.** The counting of significant figures in a number proceeds always **from the left.**

EXAMPLES.

1. State the value 3·245 in. correct to 2 significant figures. In giving the second figure, we must give the **nearest** second significant figure. This is determined by observing the value of the third figure. As before, we increase the value of the second figure by 1 if the third figure is 5 or over.

3·245 in. is 3·2 in. to 2 significant figures.

2. If the value had been 3·265 inches, the answer would have been 3·3 in.

3. ·347 correct to 2 significant figures is ·35.

4. ·0347 correct to 2 significant figures is ·035. A nought to the left of the first digit of a number has **no significance.** It only indicates place value. It indicates to us that the first digit, the 3, has a value $\frac{3}{100}$.

5. 25·898 correct to 4 significant figures is 25·90. The nought here **has significance.** In a measurement like 25,000 miles for the circumference of the earth correct to 2 significant figures, the noughts have **no significance.** They merely denote that the 25 represents 25 thousands.

Examples—*continued.*

The nought in 905, correct to 3 significant figures, has significance.

Note carefully.—An answer may be required correct either to so many decimal places or to so many significant figures. We must distinguish what is required, because a calculated answer, 27·756, correct to 2 significant figures, is 28, but, to 2 decimal places, it is 27·76.

Keep in mind that, to give any answer correct to a given number of decimal places or significant figures, we must work to one more than actually required, to enable us to give the nearest value of the last figure required.

6. Give $27 \cdot 9 \div 16 \cdot 08$ correct to the first **decimal place.** (This means we shall have to carry our calculations to the second decimal place.)

Estimated answer :—$\frac{30}{20} = 1 \cdot 5$.

$\frac{27 \cdot 9}{16 \cdot 08} = \frac{2790}{1608}$ (bringing divisor to whole number).

```
        1·73
1608)2790·
     1608
     ----
     11820
     11256
     -----
       5640
       4824
       ----
```

Answer—
1·7, correct to one decimal place.

The answer 1·7 correct to one significant figure is 2.

Using the standard form method :—

$\frac{27 \cdot 9}{16 \cdot 08} = \frac{2 \cdot 79 \times \cancel{10}}{1 \cdot 608 \times \cancel{10}}$

```
          1·73
1·608)2·79
      1·608
      -----
      1·1820
      1·1256
      ------
        5640
        4824
        ----
```

Answer—
1·7 to one decimal place.

EXERCISES.

1. (*a*) State the values supplied, to the number of significant figures indicated :—

595·54 to 4 significant figures ;
·0025 to one significant figure ;
29·99 to 2 significant figures ;
7,926·6 to 1 significant figure ;
1001 to 1 significant figure.

(*b*) Express 39·37 to 1 decimal place, 30·01 to 1 decimal place, 1·372 to 2 decimal places.

(*c*) Give approximate answers for :— 3·14159 × 27, 365·25 × 99, 199 ÷ 1·66. Set the calculations out in full, the division being given to the nearest unit.

2. The Australian wheat production for 1940/41 was estimated at 82,639,500 bushels. State this figure to the nearest thousand bushels. State if any of the noughts in the answer have significance

3. If the crop estimated in Exercise 2 were harvested from 12,454,000 acres, what would be the average yield in bushels per acre correct to one decimal place?

4. The circumference of the Moon has been stated as 6,795 miles. Express this value to the nearest thousand miles. Suppose the circumference of the Earth were also given as 24,899 miles, give approximately, to the nearest whole number, how many times it is further around the Earth than the Moon.

5. A pilot, using a map on which 1 inch represented 15·8 miles, found that two towns marked on it were 3·6 inches apart. How many miles does this map measurement represent? Express your answer to the nearest mile.

REVISION EXERCISES.

1. Calculate $\dfrac{17\cdot84 \times 3\cdot762}{81\cdot37}$ correct to 4 decimal places.

2. Give 3·14159 × 2·54 correct to 3 decimal places.

3. Find 39·37 × 12·5 correct to 3 significant figures.

4. Find 90·5 ÷ 1·76 correct to 2 significant figures.

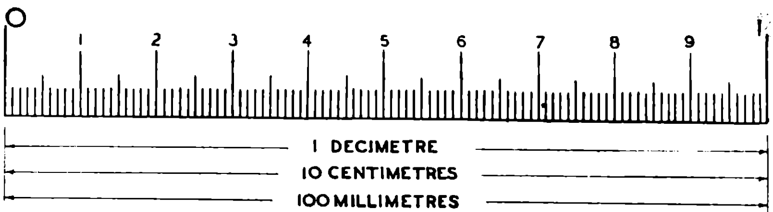
5. Multiply 6080 by 69·12 and give answer correct to nearest whole unit.

THE METRIC SYSTEM.

1. The Metric System of measurement is a decimal system. It is employed in France and most European countries, besides being used universally in scientific work.

2. **LENGTH.**—The standard Unit of Length is the **Metre.**

10 millimetres	(mm)	= 1 centimetre	(cm)
10 centimetres	(cm)	= 1 decimetre	(dm)
10 decimetres	(dm)	= 1 **metre**	(m)
10 metres	(m)	= 1 decametre	(dkm)
10 decametres	(dkm)	= 1 hectometre	(hm)
10 hectometres	(hm)	= 1 kilometre	(km).



The prefixes employed in the Metric System have meanings as follow :

milli- represents	$\frac{1}{1000}$	kilo- represents	1000 times
centi-	$\frac{1}{100}$	hecto-	100 times
deci-	$\frac{1}{10}$	deca-	10 times

Thus, a centimetre is $\frac{1}{100}$ metre, a kilometre = 1000 metres, and so on. The terms deca-, and hecto- are seldom used.

In the Metric System the idea of place value becomes very important.

Consider a measurement 1274 millimetres.

The 7 represents 70 millimetres. This is the same as 7 centimetres. Similarly, the 2 represents 2 decimetres and the 1, one metre. Hence, 1274 mm contain 1 metre 2 decimetres 7 centimetres 4 millimetres.

To convert the millimetres to any one of the other units we need only shift the decimal point to the left, thus—

$$1274 \text{ mm} = 127{\cdot}4 \text{ cm} = 12{\cdot}74 \text{ dm} = 1{\cdot}274 \text{ metres.}$$

Conversely $1{\cdot}274$ m $= 12{\cdot}74$ dm $= 127{\cdot}4$ cm $= 1274$ mm.

The Metric System—*continued.*

Given a measurement, say, 5 metres 4 cm 2 mm; to convert it to metres we need only recognize place values and insert the decimal place after the position representing metres.

5 metres 4 cm 2 mm = 5·042 metres.

The simplicity of the conversion of metric units one to the other, merely by shifting the decimal place, constitutes a great advantage in favour of the Metric System.

The metric units of length **may be compared with the British units** by remembering :—

1 inch = 2·54 cm,

1 metre = 39·37 inches,

1 kilometre = 0·62 mile (= $\frac{5}{8}$ mile approximately, or 3,280 ft.).

3. MASS.

The standard unit of mass is the **kilogramme.**

The table of mass is similar to that for length :—

10 milligrammes	(mg)	= 1 centigramme	(cg)
10 centigrammes	(cg)	= 1 decigramme	(dg)
10 decigrammes	(dg)	= 1 **gramme**	(g)
10 grammes	(g)	= 1 decagramme	(dkg)
10 decagrammes	(dkg)	= 1 hectogramme	(hg)
10 hectogrammes	(hg)	= 1 kilogramme.	(kg)

British and metric equivalents—

1 kilogramme = 2·20 lb. (approx.)

1 lb. = 453·6 g (approx.).

EXAMPLES.

(1) Convert 375 centigrammes to grammes.

Having regard to place values, the relation between cg and g and shifting the figures relative to the decimal point, we can say :—

375 cg = 3·75 g.

(2) Express 5 kg 40 grammes as grammes.

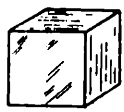
We should now be able to write straight down, **5040** gm.

4. VOLUME.

The standard unit of volume is the **litre.**

The litre is the volume of a cube each of whose sides measures 1 decimetre, i.e., a cubic decimetre (dm^3). The cubic centimetre (cm^3) is the volume that would be enclosed by a cube, each of whose sides measures 1 cm. The litre is, therefore, equivalent to 1000 cubic centimetres.

1 litre = 1·76 pints.



1 Cubic Centimetre.

EXERCISES.

1. Write down the answers to the following :—

   (*a*) How many mm are there in 15·4 cm ?

   (*b*) Convert 25·25 km to metres.

   (*c*) Express 15 kg 250 g as grammes.

   (*d*) How many cm^3 are there in 5 litres ?

   (*e*) How many pints are there in 5 litres ?

   (*f*) Express 10 metres in inches.

   (*g*) Convert 4·4 lb. to kg.

   (*h*) How many cm are there in 5·5 metres ?

   (*i*) Convert 10,000 mm to metres.

   (*j*) How many km are there in 10,000,000 metres ?

2. Add together 50 kg 150 g and 25 g 50 cg, expressing the answer in grammes.

Repeat the calculation for the subtraction of the two quantities

3. The distance between two towns is found to be 254 miles. Express this value as kilometres, to the nearest km.

4. Combats with enemy aircraft took place at 30,000 ft. or more over Britain. Express 30,000 ft. as metres, to the nearest thousand metres.

Exercises—*continued.*

5. An aircraft competing in the Schneider Trophy was supposed not to go higher than 400 metres to prevent advantage from diving. Express this height to the nearest foot.

6. A trainee pilot received instructions to fly at 4,000 ft. on a trial cross-country flight. The track he was to follow crossed a range, the altitude for which was given as 850 metres on a map he inspected. Could he say he was to fly at a sufficient altitude ?

7. The German aircraft, Junkers Ju. 52/3m. troop carrier, has an empty weight of 12,550 pounds. Express this weight in kilogrammes to the nearest kilogramme.

REVISION EXERCISES.

1. Set the following out fully :—

$$7932 + 856 + 79 + 305 + 27{,}290.$$

2. Subtract 19,356 from 100,000.

3. Add together $15 \cdot 59 + 250 \cdot 5 + 87 \cdot 405$.

4. Evaluate :—

$$3\tfrac{1}{6} + 2\tfrac{1}{2} + 1\tfrac{3}{4} + \tfrac{2}{3}.$$

5. Convert $\frac{24}{45}$ to a decimal fraction, giving answer correct to 2 decimal places.

6. Simplify :—

$$4\tfrac{3}{7} - \tfrac{5}{9} \text{ of } 4\tfrac{2}{13} + 6 \div 4\tfrac{2}{3}.$$

7. What is the least common multiple of 240, 128, 225 ?

8. Write the following in standard form :—

$$57 \cdot 25;\ 1001;\ \cdot 0259;\ \cdot 0075;\ 93{,}000{,}000.$$

9. Evaluate in full :—

$$\cdot 543 \times \cdot 0625.$$

Show your estimated answer.

10. Give $63{,}360 \div 39 \cdot 37$ to the nearest unit. Do you recognize what this represents ?

Revision Exercises—*continued.*

11. Aircraft, landing on aerodromes at high altitudes, land with a landing speed greater than at sea-level. The correction is said to be an increase of ·02 of the speed at sea-level for every 1000 ft. altitude. Correct a sea-level value of 65 m.p.h. to one at 5,000 ft.

12. The weight of the earth is said to be 6×10^{21} tons. How many kilogrammes does this represent ?

13. A cubic foot of air at sea-level is said to weigh 0·08 lb. What would be the weight of air in a large hall, if the capacity of the hall were 36,000 cubic feet ?

14. The barometric pressure reading, stated by a Weather Office on a certain day, was 30·1 inches. How many millimetres does this represent ?

15. **Add**—90247
575
6249
95
835
9

16. **Subtract**—
141,650,000 (distance of Mars from sun in miles)
92,965,000 (corresponding distance for earth)

17. $89{\cdot}32 \times {\cdot}078$.

18. $\cdot0807 \times 250$.

19. The length of the Suez Canal is 99 miles and that of Panama is 50·5 miles. Give the difference in length in miles and in kilometres (correct to the nearest km) using 1 km = ·62 mile.

20. A Hurricane aircraft carries up to 589 lb. weight in petrol. A gallon of petrol weighs 7·3 lb. What capacity does the 589 lb. petrol represent correct to the nearest gallon ?

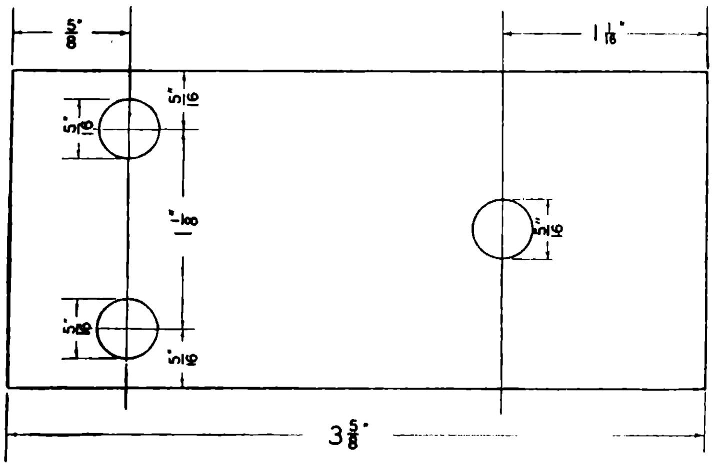
21. The areas of land surfaces on the Earth are given in sq. miles as follow :—Asia, 16,368,500 ; Africa, 11,092,750 ; Europe, 3,670,100 ; N. America, 7,623,050 ; S. America, 6,861,400 ; Australia, 3,014,050 ; Islands, 2,780,850 ; Antarctica, 3,500,000.

Find the total land area on the Earth. Give answer to nearest thousand sq. miles.

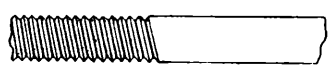
22. If the total area of the oceans on the Earth is estimated as 137,000,000 sq. miles, give an approximate answer for how many times (to the nearest whole number) the area of the oceans is greater than the area of the land masses.

Revision Exercises—*continued.*

23. The accompanying diagram represents a piece of rectangular metal drilled with 3 circular holes, each $\frac{5}{16}''$ across and placed as shown by the measurements supplied. How wide is the metal plate and how far is the centre of the hole on the right from the line passing through the centres of the other two ?



25. On a certain standard screw there are 20 threads per inch. What is the distance between each thread expressed as a decimal of an inch ?



USEFUL ARITHMETICAL RULES.

1. A number is divisible by 2 if the last figure is even.
2. A number is divisible by 3 if the sum of its digits is divisible by 3.
3. A number is divisible by 5 if the number ends with 0 or 5.
4. To multiply by 5, add 0 to the number and divide by 2.
5. To multiply by 25, add two 0's to the number and divide by 4.
6. To multiply by 50, add two 0's to the number and divide by 2.
7. To multiply by $33\frac{1}{3}$, add two 0's to the number and divide by 3.
8. To multiply by 99, add two 0's to the number to be multiplied and subtract the original number from it.

TRAINING COURSE.

GROUND DUTIES CADETS.

STAGE I.

Arithmetic. **1A/2b/1.**

REVISION OF PRELIMINARY COURSE.

1. Revise Lectures 1 and 2, 1A/1 Arithmetic.

2. Revision will include **Addition, Subtraction, Multiplication, Division** and **Tables.**

REVISION EXERCISES.

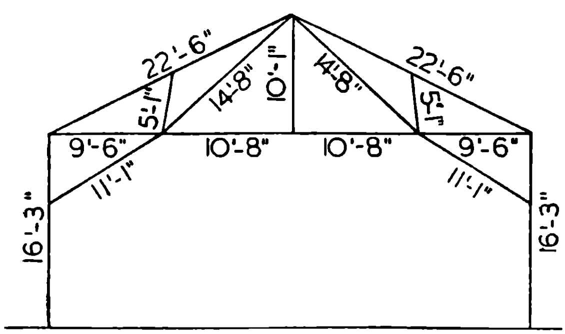
1. Which of these columns is greatest, and by how much ?

A	*B*	*C*
6162	659	5461
437	8469	7613
8916	976	1206
5137	8162	2076
1212	3517	5833
377	1892	796

2. Without re-writing these numbers, write down their total :— 767, 8148, 3212, 76, 9004, 3196.

3. A £1000 prize is shared by 37 people. How much, to the nearest shilling, does each get ?

4. 593 times a certain number is 4,230,462. Find the number



5. Fig. 1 shows the various lengths of a roof framework for a building. Find the total length of the members. If the average weight of the members is 14 lb. per foot length, find the total weight of one frame.

6. Multiply 97 by 97 (that is, "square" 97). Then divide the answer by 59 and give the answer as a whole number and fraction.

7. Multiply 3814 by 314.

Revision Exercises—*continued.*

8. Calculate :—

(*a*) Number of pints in 1 cubic foot.

(*b*) Number of cables in 5 nautical miles.

(*c*) Number of square feet in 1 square mile.

(*d*) Number of pounds in 3 tons 5 cwt. 2 qrs. 17 lb.

(*e*) Number of minutes in three-quarters of a right angle.

9. An air force tender travels 1789 miles in 9 days :—176, 205, 199, 181, 215, 164 and 156 miles on the first 7 days respectively, and 190 miles on the last day. The speedometer registered 12,905 at the start. How far did the tender travel on the eighth day, and what was the reading after the sixth day ?

10. The rate of pay for an airman is 8s. 6d. per day. He receives a meal allowance of 1s. 9d. per day, and a further sustenance allowance of 8d. per day when living off station. He is allowed 3s. 6d. per day for his wife, provided he makes an allotment to her of 5s. per day. How much would an airman receive during 1940 if he is married on the 13th April of that year, and commences living out on 24th April, and how much does his wife receive ?

REVISION OF PRELIMINARY COURSE—*continued.*

1. Revise Lectures 3 and 4, 1A/1 Arithmetic.

2. Revision will include **Multiples, Factors, Fractions, Multiplication** and **Division of Fractions.**

REVISION EXERCISES.

1. Simplify the following :—

(*a*) $\frac{3}{16} + \frac{1}{8} + \frac{5}{32} + \frac{1}{64}$.

(*b*) $1\frac{7}{8} - \frac{1}{2} + \frac{7}{16} - \frac{1}{32}$.

(*c*) $3\frac{3}{4} - \frac{1}{8} - 1\frac{5}{16} + \frac{3}{64} + \frac{1}{4}$.

2. By what quantity must the sum of $\frac{1}{3}$, $\frac{2}{5}$, $\frac{7}{9}$ be multiplied to yield $1\frac{31}{64}$?

3. When a cargo of 252 tons had been loaded in a ship, it was found that $\frac{5}{18}$ of the available tonnage was used ; what was the total tonnage ?

4. An airman travelled 56 miles by rail, 24 miles by motor car and completed the journey of 480 miles by aeroplane. Express, as fractions of the entire length of journey, the distances covered by each method of travelling.

5. Calculate the overall length of the shaft shown in Fig. 1.

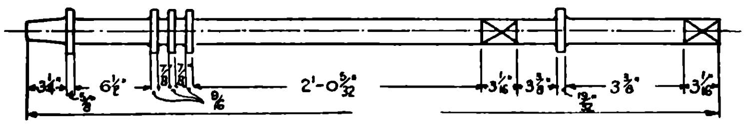


Fig. 1.

What fraction of the total length is the length of each journal ?

6. What fraction of £2 8s. is £1 6s. 8d. ?

7. Express 2 quarts 1 pint as a fraction of 36 quarts.

8. Some proportions of a shaft journal are given in Fig. 2 in terms of the shaft diameter *d*. Make similar sketches, and insert dimensions for shafts $2\frac{7}{8}$ in., $3\frac{1}{2}$ in., and $4\frac{1}{4}$ in. diameter. Give the dimensions to the nearest $\frac{1}{32}$ in.

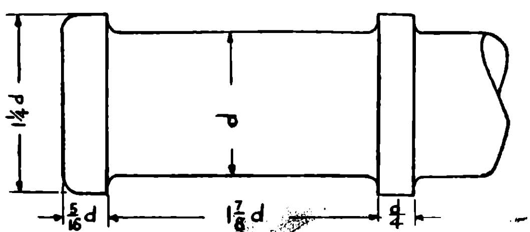


Fig. 2.

Revision Exercises—*continued.*

9. What must be added to the product of $\frac{1}{2}\frac{4}{7}$ and $\frac{4}{5}\frac{5}{6}$ to make $\frac{3}{2}$?

10. What is the difference between the product of $1\frac{1}{2}$ and $7\frac{1}{5}$ and the sum of $2\frac{3}{5}$ and $3\frac{2}{7}$?

11. The proportions of an hexagonal bolt and nut are given in Fig. 3 in terms of bolt diameter *d*. Find the dimensions for bolts $\frac{5}{8}$ in., $\frac{3}{4}$ in., $1\frac{3}{8}$ in. and $1\frac{1}{4}$ in. diameter. Draw a similar sketch showing the dimensions for a $\frac{3}{4}$-in. bolt.

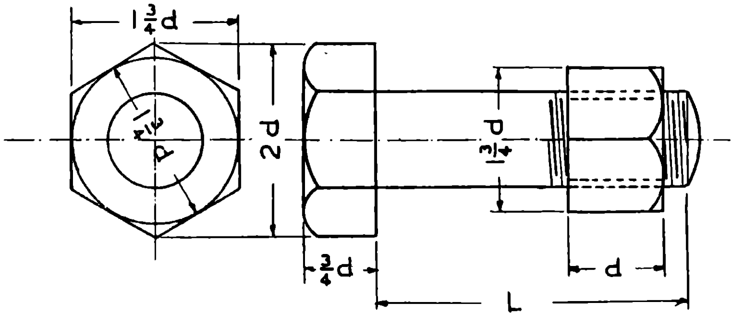


Fig. 3.

12. An aircraft tank holds 350 gallons of which $\frac{1}{7}$ is reserve. The consumption of petrol is 40 gallons per hour. What is the flying time if the reserve is not used, and what fraction of this time has been flown when there is $\frac{1}{5}$ of the total petrol still remaining in the tank ?

REVISION OF PRELIMINARY COURSE *continued.*

1. Revise Lectures 5 and 6, 1A/1 Arithmetic.

2. Revision will include **Decimals, Approximations,** and **Significant Figures.**

REVISION EXERCISES.

1. Express the following decimal quantities as vulgar fractions :—
   0·125, 0·375, 0·54, 1·875, 5·19.

2. Express the following vulgar fractions as decimal fractions :—
   $\frac{9}{16}$, $\frac{7}{8}$, $\frac{43}{100}$, $\frac{3}{50}$, $\frac{17}{20}$.

3. Calculate the following, after first expressing each decimal in the standard form :—

   (*a*) 3·405 × 9·123.

   (*b*) 159·43 ÷ 3·14.

   (*c*) 32·14 × 0·7423 ÷ 7·912.

   (*d*) (10·32 × 0·005231) ÷ 0·02076.

   (*e*) 2307 ÷ 1·354.

4. In Fig. 1, the proportions of a cup, or snap head, rivet are given. Find the dimensions when *d*, the diameter of the rivet, is (i) 0·75 in., (ii) 0·875 in., (iii) 1 in., (iv) 1·125 in.

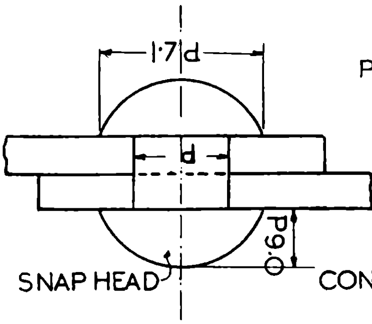


Fig. 1.

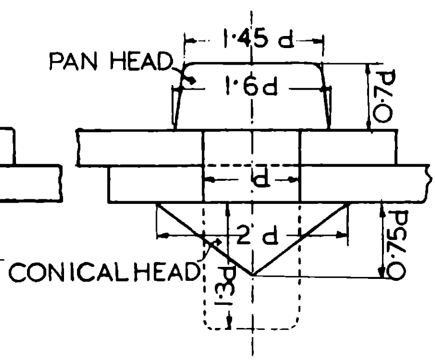


Fig. 2.

5. In Fig. 2, the proportions of a conical and pan-headed rivet are given. Find the dimensions when *d*, the diameter of the rivet is (i) 0·75 in., (ii) 0·875 in., (iii) 1 in., (iv) 1·125 in.

If the distance that a rivet projects before the head is formed is 1·3*d*, find the dimensions for the projection when *d* has the above values.

Revision Exercises—*continued.*

6. In Fig. 3, some proportions in terms of p, the pitch of the teeth of a spur wheel, are given. Make a similar sketch and mark on it the dimensions for teeth of 3·5 in. pitch.

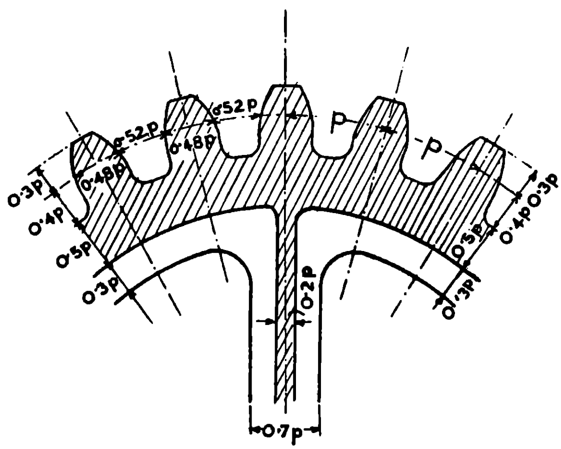


Fig. 3.

7. The mean distance of the earth from the sun has been quoted at 92,820,000 miles, and the speed of light as 186,326 miles per sec. Give the sun's distance to the nearest million miles and the speed of light to the nearest 100 miles per sec. Find, in seconds, to two places of decimals, the time taken for light to reach the earth from the sun.

8. The crank-shaft diameter of an aero-engine was given as 2·527 inches. Express the diameter correct to the nearest hundredth of an inch. Also express the diameter correct to two significant figures. Convert the given diameter to cm, and express it correct to the nearest hundredth of a cm.

9. The diameter of a piston is 48·25 cm and the stroke is 660 mm. Express these in inches (1 in. = 2·54 cm = 25·4 mm).

10. From a rod 10 feet long, portions, each 1·9 in. in length, are cut off. How many pieces can be cut off, and what is the length of the remaining piece, if the thickness of each saw cut is 0·18 in. ?

11. Which is greater, $\frac{7}{16}$ inch or 0·475 inch ? Work it out in two ways. Which way is easier ?

12. 3·7, 4·835, 6·875. Add these and change the answer to a fraction. Now change the three given numbers to fractions, add them and change the answer to a decimal.

13. Wrought iron gas tubes of 1 inch-bore weigh 1·79 lb. per foot length. Find the weight of 1 mile of such tubing.

REVISION OF PRELIMINARY COURSE—*continued.*

1. Revise Lecture 7, 1A/1 Arithmetic.
2. Revision will include the **Metric System.**

REVISION EXERCISES.

1. Convert the following lengths to centimetres. Express each answer correct to two places of decimals :—5 in., $3\frac{1}{16}$ in., $\frac{1}{8}$ in., 3 ft. 4 in., 1 yard 0 ft. 2 in., $1\frac{7}{8}$ in.

2. Convert the following lengths to inches. Express each answer to the nearest sixteenth of an inch :—10·1 cm, 143 mm, 5·31 cm, 1·85 cm, 28 mm, 0·46 m.

3. The weight of an aircraft is 18,000 lb. Give this weight in kilogrammes to the nearest hundred kilogrammes.

4. How many tons are there in 6000 kilogrammes ? Give the answer to the nearest ton.

5. An aircraft is flying at 300 m.p.h. It departs on an operational cruise at 10 a.m. and returns at 2.45 p.m. How far in kilometres has it flown ? Give the answer to the nearest km.

6. If $\frac{5}{16}$ in. holes are to be drilled in metal, but only an 8 mm drill is available, would the holes be too large or too small ?

7. A petrol tank holds 200 gallons. How many litres is this ? Give answer to the nearest litre. If an aircraft, travelling at 400 km per hour, empties this tank in $4\frac{1}{2}$ hours, calculate the petrol onsumption in miles per gallon and litres per kilometre.

RATIO AND PROPORTION.

1. A **ratio** is a comparison between two quantities **of the same kind.** It states the **number** (whole or fractional) **of times** the first quantity contains the second.

If a room is 30 ft. long and 25 ft. broad, the ratio of its length to its breadth is $\frac{30}{25}$, or as generally written, 30 : 25, and read "30 to 25".

Or again, if we say the ratio of the circumference of a circle to its diameter is $\frac{22}{7}$ (or, 22 : 7) we mean the circumference contains the diameter $\frac{22}{7}$ or $3\frac{1}{7}$ times.

A ratio may be reduced to its lowest terms by cancelling any common factor.

Thus $30 : 25 = 6 : 5$.

If the breadth of the room, in the example above, had been 15 ft., the required ratio would have been 30 : 15, i.e., 2 : 1, or just simply 2.

A ratio may be expressed also as a decimal, thus; 6 5 could be replaced by 1·2 : 1, or just 1·2.

EXAMPLES.

(1) What is the ratio of 1 inch to 1 mile ?

The two quantities must be expressed in **the same units.**
Bring the mile to inches.
1 mile = 63,360 inches
Ratio 1 inch to 1 mile

$$= \frac{1}{63{,}360}$$

(2) What is the ratio of 1 cm to 1 inch ?

The two quantities have to be of the same units.

Solution 1—

Bring 1 cm to inches.

$$1 \text{ cm} = \cdot 3937 \text{ inches.}$$

$$\text{Ratio} = \frac{\cdot 3937 \text{ inch}}{1 \text{ inch}} = \cdot 3937.$$

Solution 2—

Bring 1 inch to cm

$$1 \text{ inch} = 2\cdot 54 \text{ cm}$$

$$\text{Ratio} = \frac{1 \text{ cm}}{2\cdot 54 \text{ cm}} = \frac{1}{2\cdot 54}$$

It must be realized that more than one result is often possible in these examples. The one which gives the simplest result should be selected.

2. PROPORTION.

Four quantities are said to be **in proportion** when the ratio of the first to the second is equal to the ratio of the third to the fourth. Thus, since the ratio 4 : 8 is equal to the ratio 16 : 32, the four terms, 4, 8, 16 and 32 are said to be in proportion. This fact can be expressed, 4 : 8 = 16 : 32 or, 4 : 8 : : 16 : 32. The symbol : is equivalent to a division sign and : : to an equality sign, so that 4 8 :: 16 32 is the same as $\frac{4}{8} = \frac{16}{32}$.

Notice that $4 \times 32 = 8 \times 16$.

The first and last terms of a proportion are called **the extremes,** and the second and third terms, **the means.** Hence, we can say the product of the extremes = the product of the means. The rule is useful for finding a missing term in a proportion. Thus, suppose we replace a missing term by x :—

$$4 : x = 16 : 32.$$

Then, $4 \times 32 = 16 \times x$, so that x must be 8.

Or again if 4 $8 = 16$ x then, $4 \times x = 8 \times 16$.

x can be seen to be 32.

N.B.—Although ratios always refer to two quantities of the same kind, yet the two sides of a proportion need not necessarily refer to quantities of the same kind. Thus the four terms 3-lb., 5-lb., 12s. and 20s. are in proportion.

3. DIRECT AND INVERSE PROPORTION.

If an aircraft travels at a uniform speed of 100 m.p.h., it will travel 100 miles in one hour; 200 miles in 2 hours; 300 miles in 3 hours and so on. The distance it travels is then **directly proportional** to the time taken, or distance **varies directly** as time. As the time increases, the distance travelled increases in the same ratio.

e.g.—If time increases from 1 hour to 3 hours, i.e., in ratio 1 3, the distance travelled is increased in the ratio of 1 3.

Again, a journey of 600 miles, to be travelled by a plane at 300 m.p.h., would take 2 hours; at 200 m.p.h., 3 hours; at 100 m.p.h., 6 hours; and so on. Thus, as the speed **decreases,** the time taken **increases** proportionally. The time taken is **inversely proportional** to the speed, or time **varies inversely** as speed. If speed decreases from 300 m.p.h. to 200 m.p.h., i.e., in ratio 300 : 200 or 3 : 2, time increases from 2 hours to 3 hours, i.e., in ratio 2 3. The ratio 2 : 3 is the inverse of the ratio 3 2.

Note.—In the preliminary course, examples of direct and indirect proportion occurred in the Science section; for example, in Newton's Second Law, there is an instance of direct proportion. The acceleration produced in a given mass is directly proportional to the force causing it. Similarly in Newton's Law of Universal Gravitation, an instance of inverse proportion arose. The force due

3. Direct and Inverse Proportion—*continued.*

to gravitation existing between two masses is inversely proportional to the **square** of their distance apart. This means that, if the distance apart is doubled, the force is one-**quarter** of its former value; if it is trebled, the force is one-**ninth** of its former value, and so on.

Other examples will be found elsewhere in these courses.

EXAMPLES.

(1) When using a spanner of certain length to tighten a nut, the pull of the spanner is made **directly proportional** the size of the nut.

If a force of 10 lb. is exerted to tighten a $\frac{3}{8}$-inch nut, what force must be exerted to tighten a $\frac{5}{16}$-inch nut?

10 lb. are used for $\frac{3}{8}$-inch nut

x lb. are used for $\frac{5}{16}$-inch nut

$$x = 10 \times \frac{\frac{5}{16}}{\frac{3}{8}} \text{ lb.}$$

$$= \frac{\overset{5}{\cancel{10}} \times 5 \times \cancel{8}}{\underset{2}{\cancel{16}} \times 3} \text{ lb.}$$

$= 8\frac{1}{3}$ lb. (Note that force is **smaller** for a **smaller** nut.)

N.B.—This variation in the pull on a spanner is necessary to prevent bursting the nut.

(2) If the temperature remains the same, the volume of a gas is **inversely proportional** to its pressure.

If the volume of an airship is 5 million cubic feet, when the pressure is 15 lb. per square inch, what would be the volume at the same temperature if the pressure is decreased to 10 lb. per square inch?

5,000,000 c. ft. is volume at 15 lb. per square inch pressure.

x c. ft. is volume at 10 lb. per square inch pressure.

$$x = 5{,}000{,}000 \times \frac{15}{10} \text{ c. ft.}$$

$$= \frac{\overset{2{,}500{,}000}{\cancel{5{,}000{,}000}} \times \overset{3}{\cancel{15}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{10}}} \text{ c. ft.}$$

$= 7{,}500{,}000$ c. ft. (Note that volume is **greater** for a **smaller** pressure.)

N.B.—This expansion is restricted in practice by suitable construction of the airship.

Examples—*continued.*

(3) Other conditions being equal, the lifting force on an aircraft is **proportional to the square** of its speed.

If the lift is 6 tons when a bomber is flying at 180 m.p.h., what is its lift if the speed is suddenly reduced to 160 m.p.h. ?

6 tons is lift at 180 m.p.h.

x tons is lift at 160 m.p.h.

$$x = 6 \times \frac{(160)^2}{(180)^2} \text{ tons.}$$

$$= \frac{\overset{1}{\cancel{6}} \times \overset{8}{\cancel{160}} \times \overset{16}{\cancel{160}}}{\underset{\underset{3}{\cancel{18}}}{\cancel{180}} \times \underset{9}{\cancel{180}}} \text{ tons.}$$

$$= \frac{128}{27} \text{ tons.}$$

$$= \mathbf{4 \cdot 7 \text{ tons.}}$$

N.B.—For level flight, this loss of lifting force would be counteracted by adjustment of the controls.

EXERCISES.

1. What is the ratio of 1 km to 1 mile ?
2. What is the ratio of 1 chain to 1 mile ?
3. What is the ratio of 1 lb. to 1 g. ?
4. What is the ratio of 1 cm to 1 inch ?
5. What is the ratio of 4 miles to 60 miles ?
6. What is the ratio of 120 minutes to 18 degrees ?
7. What is the ratio of 1 pint to 1 litre ?
8. What is the ratio of 1 gallon to 100 litres ?
9. Are the following sets of terms in proportion ?

   7, 5, 21, 15 ;

   8, 9, 40, 44 ;

   3 ft., 10 ft., 1 shilling, 3s. 4d.

10. The first three of four terms in proportion are 14 cwt. 8 lb., 6 cwt. 8 lb., and £27 1s. 9d., what is the fourth ?

11. Suppose two quantities a and b are directly proportional. If the value of b is altered to $\frac{2}{3}$ of its original value, what change would take place in the value of a ? If the proportionality was altered to an inverse proportion, what change would take place in a for the same change in b above ?

Exercises—*continued.*

12. If the pull on a spanner to tighten a certain size of nut is directly proportional to the length of the spanner, find the pull exerted if the spanner's length is—

(i) 8 in.,

(ii) $9\frac{3}{4}$ in.,

(iii) $5\frac{7}{8}$ in.,

given that the pull on a 10-inch spanner is 16 lb.

13. Up to certain limits, the extension of a bar of steel is directly proportional to the force causing the extension.

If a force of 3 tons causes an extension of 0·002 in., then find the extension caused by a force of—

(i) 5·2 tons,

(ii) 1·9 tons,

(iii) 3000 lb.

What would be the force necessary to cause an extension of—

(i) 0·0016 in.,

(ii) $\frac{1}{64}$ in. ?

14. If the lifting force on an aircraft is 15 tons when flying at 200 m.p.h., what lift would be obtained if the speed were suddenly increased to 250 m.p.h. ? What would happen to the aircraft ?

15. At a certain moment, a stick 4 ft. high throws a shadow 6 ft. long. At the same time a tower throws a shadow 486 ft. long. Find the height of the tower.

AVERAGES—PERCENTAGES.

1. AVERAGES.

(1) When the sum of a number of quantities of the same kind is divided by the number of the quantities, the result is called the **average** (or **mean**) of the given quantities.

$$\text{i.e., Average} = \frac{\text{Sum of the given quantities}}{\text{Number of quantities}}$$

EXAMPLE.

The ages of 5 men were 89, 83, 80, 79, and 74 years respectively, what was their average age ?

The sum of the 5 ages = 405 years.

$$\text{Average age} = \frac{405}{5} = \mathbf{81\ years.}$$

(2) Given the average value of a given number of quantities, their sum can be found by multiplying their average value by the number of them.

i.e.—Sum of the given quantities = their average $\times$ the number of them.

EXAMPLE.

Over a period of 4 years, the United Kingdom had under construction an average of 412 ships per year. The total number under construction in 4 years = $412 \times 4 =$ **1648.**

2. PERCENTAGES.

A percentage is a fraction of which the denominator is 100.

The words **per cent.** owe their origin to the Latin **per centum,** meaning " for every hundred ".

Thus, if we were to say that 71 per cent. of the candidates for a certain examination passed the test, we should mean that $\frac{71}{100}$, i.e., 71, out of every hundred of those who sat, passed.

The symbol % is employed for per cent.

71 per cent. = 71 % = $\frac{71}{100}$.

EXAMPLES.

(1) What fraction is equivalent to 75 % ?

$$75\ \% = \tfrac{75}{100} = \tfrac{3}{4},$$

i.e., express the per cent. given as a fraction with denominator 100. Cancel where necessary. (N.B.—100 % is equivalent to 1.)

(2) What percentage is $\frac{3}{5}$?

$$\tfrac{3}{5} = \tfrac{3}{5} \text{ of } 1 = \tfrac{3}{5} \text{ of } 100\ \% = 60\ \%,$$

i.e., to convert a fraction to a percentage simply multiply the numerator by 100.

Examples—*continued.*

(3) **What** is 5 % of 240 ?

$$5\% = \tfrac{5}{100} = \tfrac{1}{20}.$$
$$\tfrac{1}{20} \text{ of } 240 = \mathbf{12}.$$

(4) What per cent. of 1 mile is 440 yards ?

Express the 440 yards as a fraction of the mile.

$$\frac{440 \text{ yards}}{1 \text{ mile}} = \frac{440}{1760} = \frac{1}{4}.$$

$\frac{1}{4}$ as a per cent. $= \frac{1}{4} \times 100 = \mathbf{25}\ \%$.

(5) 97 is 10 % of what number ?

If 97 = 10 % of a number, it equals $\frac{10}{100}$ (or $\frac{1}{10}$) of that number.

∴ the number must equal 97 × 10 = **970.**

N.B.—The following example should be carefully observed.

(6) What is the ratio of 1 kilometre to 1 statute mile? Express the ratio in both the fractional and decimal form. What per cent. of the statute mile is the kilometre ?

$$\text{Required ratio} = \frac{1 \text{ kilometre}}{1 \text{ statute mile}}$$
$$= \frac{3280}{5280}$$
$$= \frac{\mathbf{41}}{\mathbf{66}} \qquad (\text{or } 41 \quad 66)$$

The decimal form of $\frac{41}{66}$ is 0·62 (to two decimal places).

The required per cent. equals either $\frac{41}{66} \times 100$, or $0{\cdot}62 \times 100$.

i.e., it is 62 % to the nearest per cent.

From these answers, we can see the connections between the terms ratios, fractions, decimals, percentages. The ratio of 1 kilometre to 1 mile is 41 : 66. A kilometre is $\frac{41}{66}$ (about $\frac{5}{8}$) of a mile, 0·62 mile, or 62 % of a mile.

3. PERCENTAGE ERRORS.

(1) Consider a length 2·5 inches, given correct to the nearest tenth of an inch.

From what is given, all we can say is that the true value lies between 2·45 and 2·55 inches, and the greatest error 2·5 inches can have is 0·05 inch, either above or below.

The given value could be expressed—

$$2{\cdot}5\ (\pm\ 0{\cdot}05),$$

where the number ± 0·05 is called **the Absolute Error.**

3. Percentage Errors—*continued.*

Errors may also be indicated by a fraction, the numerator being the absolute error, and the denominator the value to which the absolute error refers. In this way we obtain **the relative error.**

The relative **error** in this case $= \dfrac{0 \cdot 05}{2 \cdot 5} = \dfrac{5}{250} = \dfrac{1}{50}$.

A **relative** error may be expressed as a percentage, giving what is known **as the percentage error.**

The percentage error in this case $= \frac{1}{50} \times 100 = 2\ \%$,

(2) Consider as another example, a value of $0 \cdot 25$ given correct to two significant figures. Following the procedure as above, we could express the given value as $0 \cdot 25\ (\pm\ 0 \cdot 005)$.

The relative error $= \dfrac{0 \cdot 005}{0 \cdot 25} = \dfrac{5}{250} = \dfrac{1}{50}$

The percentage error $= \frac{1}{50} \times 100 = 2\ \%$.

(3) Suppose the value given in the first example had been $12 \cdot 5$ inches. The absolute error would still be $\pm\ 0 \cdot 05$, but the relative error would be

$$\frac{0 \cdot 05}{12 \cdot 5} = \frac{5}{1250} = \frac{1}{250} \text{ and}$$

the percentage error would equal $\frac{1}{250} \times 100 = 0 \cdot 4\ \%$.

It should be appreciated from the exercises given that the significance of an error is better grasped by giving the relative (or percentage) error than by giving the absolute error. An error of one mile in distance of 50 miles represents a percentage error of $\frac{1}{50} \times 100 = 2\ \%$; in 500 miles it is $\frac{1}{500} \times 100 = 0 \cdot 2\ \%$; whilst in 5000 miles it is only $\frac{1}{5000} \times 100 = 0 \cdot 02\ \%$.

EXERCISES.

1. What fractions are equivalent to $40\ \%$, $150\ \%$, $12\frac{1}{2}\ \%$, $2\frac{1}{2}\ \%$, $66\frac{2}{3}\ \%$.

2. Express as percentages :—$\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{3}{5}$, $\frac{7}{10}$, $\frac{5}{8}$, $2\frac{1}{2}$, $\cdot 75$.

3. Find the values of :—3 % of 150, $3\frac{1}{3}$ % of £30, 5 % of £5 10s., 10 % of 1 metre expressed in centimetres, $6\frac{1}{4}$ % of 112 lb.

4. The following is a set of temperature readings taken hourly at a Weather Office :—40°, 42°, 45°, 50°, 55°, 58°, 57°, 54°, 52°, 51°, 49°, 47°. What is the mean value of these observations ?

5. The average weight of a rowing crew, consisting of 8 members, is 10 st. $10\frac{1}{2}$ lb. What is their total weight ? The weight of the coxswain is 9 st. $6\frac{1}{2}$ lb. What would the average weight be if his weight were included with the others ?

Exercises—*continued.*

6. A aircraft fitter recorded his daily inspection periods. If the entries for five consecutive days were 35 minutes, 40 minutes, 25 minutes, 30 minutes, 35 minutes, and that for the sixth day was nil, due to absence of aircraft, what was his average daily inspection time for the period quoted ?

7. If you were told that an airscrew, driven by a 1000 horse-power engine wasted 250 horse-power, what percentage of the available horse-power was utilized ?

8. In the Revision Exercises to Lecture 1, 1A/1 Arithmetic, find :—

(*a*) In question 3, the percentage increase in the population.

(*b*) In question 5, the percentage of cases not interned.

9. Two results are expressed as follows :

$$38{\cdot}4 \ (\pm\ 0{\cdot}05)$$
$$0{\cdot}003840 \ (\pm\ 0{\cdot}0000005).$$

Calculate the percentage errors in each case.

10. A value was given as 63·29. If there were a possible error of 2 in the fourth significant figure, find the absolute error, the relative and percentage errors.

SQUARE ROOT.

1. POWER AND ROOT.

(i) $5 \times 5 = 25$.

25 is then called the square of 5. It may be written 5^2, read "five squared" or five to the **power** 2.

(ii) Because $5 \times 5 = 25$, 5 is the **square root** of 25,

$$\text{or } 5 = \sqrt{25}.$$

(iii) Similarly, since $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$, then $\frac{1}{3}$ is the square root of $\frac{1}{9}$,

$$\text{or } \tfrac{1}{3} = \sqrt{\tfrac{1}{9}}.$$

$$\text{Also } \tfrac{1}{9} = (\tfrac{1}{3})^2.$$

(iv) When taking the square root of decimal, care must be taken with the position of the decimal point.

$$\text{Thus } (0{\cdot}15)^2 = 0{\cdot}0125.$$

$$\text{Hence } \sqrt{0{\cdot}0125} = 0{\cdot}15.$$

N.B. It will be noticed, as a check, that the square root of a fraction is **greater** than the fraction itself.

2. TO FIND A SQUARE ROOT.

(1) Find the square root of 100,489:

$$\begin{array}{r|l}
 & \;3\;\;1\;\;7 \\
\hline
3 & 10{,}04{,}89{\cdot}00{,}00{,}00 \\
 & \;9 \\
\hline
61 & 104 \\
 & \;\;61 \\
\hline
627 & \;\;4389 \\
 & \;\;4389 \\
\hline
\end{array}$$

Answer: 317

Method.

(*a*) Group the numbers in pairs to the left and to the right of the decimal point.

(*b*) Find the number whose square is equal to or just less than the number formed by the first pair on the left, e.g., 10 above.

(*c*) Write this number in the answer (above line) and put its square below the 10.

(*d*) Subtract.

(*e*) Bring down the next pair of numbers.

(*f*) Double the answer, which was already obtained and set down above the line, and write this product in left-hand column, e.g., 6.

(*g*) Find the figure (in this case, 1) which, when multiplied by the number formed by placing the same figure after the 6, will give a product equal to or just less than 104.

2. **To find a Square Root**—*continued.*

(*h*) Continue steps (*d*) to (*g*) noting that, in the given example, 31 doubled becomes 62. 7 is then the figure which is placed after 62, i.e., 627, and is the multiplier to give the product 4389.

(2) Find square root of 9·3025:

$$
\begin{array}{r|l}
 & 3\cdot\ 0\quad 5 \\
\hline
3 & 9\cdot 30,25 \\
 & 9 \\
\hline
60 & 30 \\
 & 0 \\
\hline
605 & 30,25 \\
 & 30,25 \\
\hline
\end{array}
$$

Answer: 3·05.

(3) Find square root of 2, correct to 3 decimal places:

$$
\begin{array}{r|l}
 & 1\cdot 4\quad 1\quad 4\quad 2 \\
\hline
1 & 2\cdot 00,00,00,00 \\
 & 1 \\
\hline
24 & 1\ 00 \\
 & 96 \\
\hline
281 & 400 \\
 & 281 \\
\hline
2824 & 11900 \\
 & 11296 \\
\hline
28282 & 60400 \\
 & 56564 \\
\hline
 & 3836 \\
\hline
\end{array}
$$

Answer: 1·414.

(4) (*a*) When finding the square root of fractions, it is usually advisable to express the fraction as a decimal and then proceed as just shown. Sometimes, however, the factor method can be used. Examples :-

(i) $\sqrt{1\frac{5}{9}} = \sqrt{1\cdot 55\dot{5}}$

Proceed by square root method.

(ii) $\sqrt{6\frac{1}{4}} = \sqrt{\frac{25}{4}}$

$= \frac{5}{2}$

$= 2\frac{1}{2}$.

(*b*) Often factors may be easily taken out of a number whose square root is required, so that only a simple root requires to be evaluated.

2. **To find a Square Root**—*continued.*

Example 1.

$$\sqrt{98 \times 27} = \sqrt{2 \times 7 \times 7 \times 3 \times 3 \times 3}$$
$$= 21\sqrt{6}$$

$\sqrt{6}$ is either known or easily found. Its value is then multiplied by 21 to give the required square root.

(*c*) Commonly used square roots :—

$$\sqrt{2} = 1{\cdot}414$$
$$\sqrt{3} = 1{\cdot}732$$
$$\sqrt{5} = 2{\cdot}236$$
$$\sqrt{6} = 2{\cdot}449$$
$$\sqrt{10} = 3{\cdot}162$$

(*d*) Division by a decimal square root can often be avoided by the following device, known as **rationalizing the denominator.**

Example 2.

Given $\sqrt{2} = 1{\cdot}414$, find the value of $\frac{1}{\sqrt{2}}$.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{2}}{2}$$
$$= \frac{1{\cdot}414}{2}$$
$$= 0{\cdot}707.$$

EXERCISES.

1. Evaluate :—

(*a*) $\sqrt{6}$ (*d*) $\sqrt{0{\cdot}0289}$ (*g*) $\sqrt{0{\cdot}25}$

(*b*) $\sqrt{2{\cdot}5}$ (*e*) $\sqrt{0{\cdot}00113}$ (*h*) $\sqrt{729}$

(*c*) $\sqrt{165{\cdot}4}$ (*f*) $\sqrt{1{\cdot}0011}$ (*i*) $\sqrt{10{\cdot}391}$

2. Using $3 = 1{\cdot}732$, find the value of :—

(*a*) $\sqrt{12}$ (*b*) $\sqrt{\tfrac{1}{3}}$.

3. The force of gravitation between two bodies is inversely proportional to the square of their distance apart.

If the force is 1 lb. weight when the bodies are 3 inches apart, how far apart are they when the force is 7 lb. weight ?

4. The drag, or resistance to motion, of an aircraft is proportional to the square of its speed. If the drag is 500 lb. when the speed is 210 m.p.h., what speed would give a drag of 420 lb. ?

STATISTICAL GRAPHS.

1. Consider the following set of figures, which gives values of the atmospheric pressure at various altitudes.

Height above sea level in feet	0	5000	10,000	15,000	20,000	25,000	30,000
Atmospheric pressure measured in millibars	1016	845	690	573	466	370	302

From this table, it can be readily seen that the atmospheric pressure **decreases** as the altitude **increases,** but **how** the pressure varies with gradually increasing altitude, is not so readily apparent. Until the manner, in which these changes take place, is known, it is difficult to estimate with certainty the pressure at altitudes other than those stated. However, the variations of pressure with altitude may be clearly observed if they are represented in a diagram or **graph.**

2. CONSTRUCTION OF A GRAPH.

The procedure in graphically representing the figures tabulated above is as follows :—

(1) Two straight lines, or **axes** as they are called, are first drawn at right angles to each other, one horizontally across the page and the other vertically up the page. The given altitudes are marked to scale along the horizontal line, and the atmospheric pressures, also represented to scale, along the vertical line.

(2) The scales should be chosen before the axes are actually drawn. This will ensure that the graph is placed to advantage in the space available, and that all values involved are provided for.

In the table supplied, six altitude readings above sea-level are given, each differing from that preceding it by 5,000 feet. For the average page, a suitable scale would be :—

One unit along the horizontal line represents 5,000 ft., i.e., a horizontal axis six units long would be required.

The greatest value given for the atmospheric pressure is 1016 millibars. If then 1 unit along the vertical line represented 200 millibars, all values of atmospheric pressure supplied could be represented along a vertical axis just over 5 units long.

2. **Construction of a Graph**—*continued.*

(3) It is customary to employ **squared paper** to facilitate the construction of graphs. The commonest form of squared paper is that which is ruled horizontally and vertically, with straight lines one-tenth inch apart. Every tenth line is ruled more heavily than the others.

Using such paper, it is only necessary to rule in the horizontal and vertical lines of the selected lengths at the most suitable position on the page.

In the diagram below, they are drawn along the bottom and extreme left lines respectively.

N.B.—The point at which the two axes intersect is called the **origin.**

(4) The lines are labelled as shown :—

Height above sea-level in feet (horizontal axis),
Pressure in millibars (vertical axis).

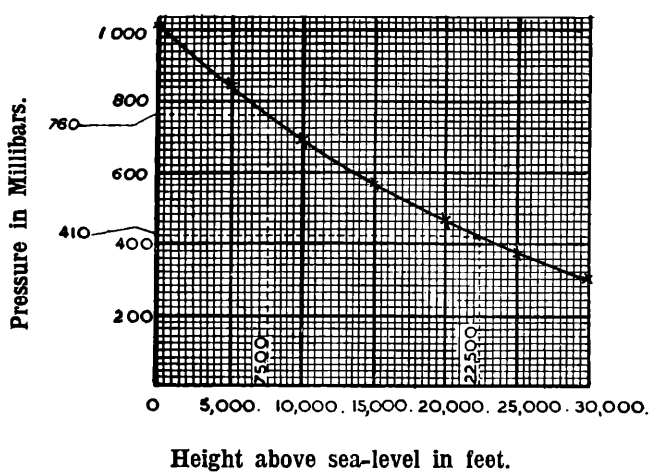
(5) One unit along the horizontal axis represents 5,000 feet, and so the intervals along it are marked as shown.

Since each unit along the vertical axis represents 200 millibars, the intervals are also marked as shown.

VARIATION OF ATMOSPHERIC PRESSURE WITH ALTITUDE.

Scale—Horizontal Axis: 1 unit = 5,000 ft.

Vertical Axis: 1 unit = 200 millibars.



Notice.— (i) The graph is given a title as shown at the top of the diagram.

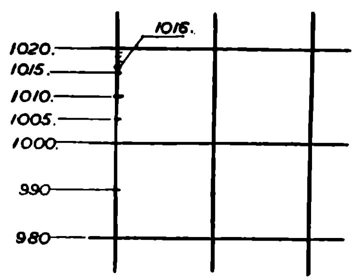
(ii) The scales are clearly marked.

(iii) A graph is normally made to be used and so should be a model of neatness and accuracy.

2. **Construction of a Graph**—*continued.*

(6) Referring to the tabulated figures, it is seen that at sea-level the atmospheric pressure is 1016 millibars. Sea-level is represented on the horizontal line at the origin. The value 1016 millibars is found on the vertical line passing through the origin by employing the scale marked on it. The position representing this pressure is marked, when located, with a small neat cross, the centre of the cross being at the position determined.

Note.—In locating the value 1016, it should be noted that each tenth of an inch on the vertical axis represents 20 millibars. Consequently, the position of the 16 millibars beyond the 1000 mark can only be **estimated.** Its position may be fixed with a fair degree of accuracy from the fact **that 16 is a** trifle greater than $\frac{3}{4}$ of 20. The following enlarged drawing should make the process clear :—



(7) When the altitude is 5000-ft., the pressure is 845 millibars The altitude is located on the horizontal axis and the corresponding pressure is plotted on the vertical line passing through the 5000-ft. mark. Its position is marked with a neat cross.

The value 840 millibars is readily located on the vertical line by utilizing the scale for the vertical axis. The remaining 5 millibars can be estimated as $\frac{1}{4}$ of a tenth of an inch.

(8) The remaining pairs of figures supplied in the **table** are treated similarly.

(9) The series of crosses obtained will **not** lie along a straight line, but, if they have been carefully located, will lie very closely to a **curved line.**

Practice is needed to draw a smooth curved line passing through a series of points, but it is important to acquire the art. It is advisable to turn the page so that the hand drawing the curve is inside the curve, i.e., on the concave side.

3. USES OF THE GRAPH SO CONSTRUCTED.

It should readily be appreciated that the graph conveys a clearer impression of the changes of atmospheric pressure with altitude than the figures alone do. The decrease of pressure with increase in altitude is clearly represented. It can be seen at a glance that the pressure does not drop off uniformly. It can be seen too that the decrease at higher altitudes is not so rapid as at lower altitudes.

Once the graph has been constructed, the pressure at any height, for the range of altitudes covered, is quickly read off. Thus, to find the pressure at 7,500 ft., the altitude is located on the horizontal axis and a vertical line is drawn upwards to meet the curve. From the point of intersection with the curve, another straight line is drawn horizontally across the page to the pressure scale at the side. It should be seen that the pressure here concerned is 760 millibars approximately.

Similarly, the altitude corresponding to a given pressure, within the range covered, can be read off. Thus, to find the altitude at which the pressure is 410 millibars, the given pressure is located on the vertical axis. A line is drawn horizontally from this position to meet the curve, and another line is drawn vertically downwards, from the point of intersection on the curve, to the altitude scale. It should be seen that the corresponding altitude is 22,500 feet, approximately.

4. FURTHER EXAMPLES.

Graphs are extensively used in practice. Newspapers often represent, in a graphical form, facts they wish to convey to readers. By means of graphs, business men may watch the trend of their turnover, military authorities the rate of enlistments, and weather experts the variations of temperature throughout the day. Where calculations involve the repeated use of a formula, it is often convenient to plot the equation in the form of a graph for the range of values involved in the work in hand.

Many machines are constructed to draw graphs automatically. Thus, there are self-recording barometers (or barographs), recording voltmeters, recorders that register the amount of water supplied at any time, indicator diagrams that show the pressure in an engine cylinder at any point of the stroke, and so on.

The following are a few worked examples of graphs.

Example 1.

If 76 statute miles = 66 nautical miles, construct a graph for converting statute miles to nautical miles for values of statute miles from 0 to 500.

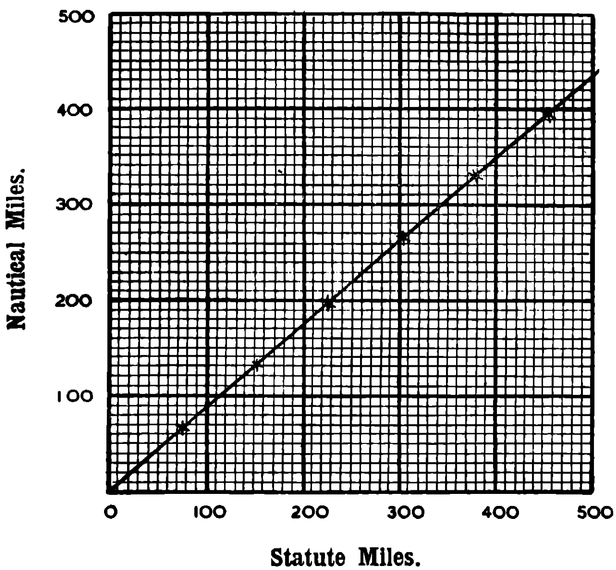
Since, 76 statute miles = 66 nautical miles,
then 152 statute miles = 132 nautical miles,
and, 228 statute miles = 198 nautical miles, &c.

4. Further **Examples**—*continued*.

These three pairs of values are sufficient to show that the graph is a straight line passing through the origin (0 statute miles = 0 nautical miles). The graph obtained is as shown.

CONVERSION OF STATUTE MILES TO NAUTICAL MILES.

Scale—Horizontal Axis: 1 unit = 100 statute miles.
Vertical Axis: 1 unit = 100 nautical miles.



From this graph, pairs of values, other than those plotted, can readily be read off. Thus, it is quickly found that 500 statute miles correspond to 434 nautical miles, and so on for other numbers of statute miles within the range plotted.

Such a graph is often called a "**ready-reckoner**" graph.

Example 2.

Suppose a certain recruiting depot had the following numbers of successful applicants each month throughout a year. Draw a graph to show the fluctuation of successful applicants from month to month.

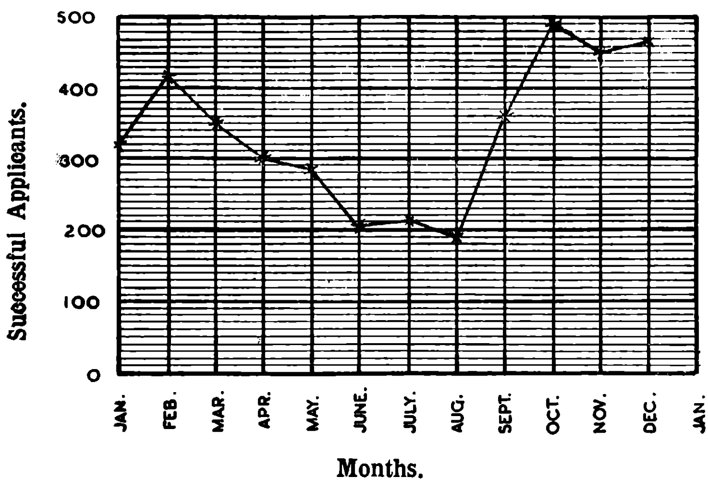
Month	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Successful Applicants ..	320	415	350	300	285	203	210	190	360	490	450	465

4. **Further Examples**—*continued.*

SUCCESSFUL APPLICANTS, YEAR ....

Scale—Horizontal Axis: 1 unit = 1 Month.

Vertical Axis: 1 unit = 100 Applicants.



The points are joined by a series of straight lines. At a glance it can be seen when recruiting figures were high, and when they were low. The most successful month and the worst can at once be picked out with ease.

Example 3.

The following figures were obtained as a result of trials on a Diesel-engined vessel, the indicated horse-power at various speeds being shown. Find, from a curve, the horse-power required at 10 knots.

Speed in Knots	7·7	9·0	9·65	10·8	11·1
Indicated H.P.	800	1200	1440	2200	2600

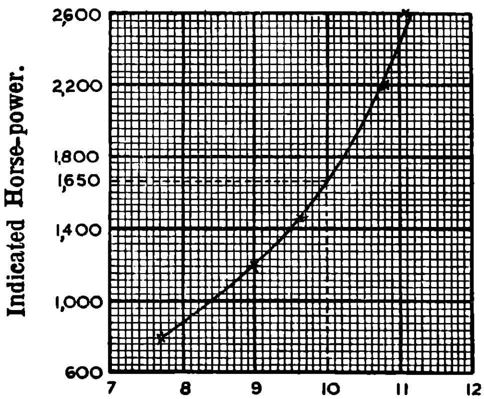
Note.—Up to the present it has been assumed that the point of intersection of the horizontal and vertical axes should represent zero reading on each scale. This is not always necessary. In many cases, much space can be saved on the page by allowing the point of intersection to represent more convenient values. On the following graph it represents 7 knots on the horizontal axis and 600 h.p. on the vertical axis.

4. **Further Examples**—*continued.*

INDICATED HORSE-POWER AT VARIOUS SPEEDS.

Scale—Horizontal Axis: 1 unit = 1 knot.

Vertical Axis: 1 unit = 400 h.p.



Speed in Knots.

At 10 knots, the horse-power required = 1660 h.p. approximately.

5. OBSERVE CAREFULLY.

(1) The accuracy of answers obtained from graphs will depend upon how carefully the graphs are constructed. The scales selected should be as large as the page will allow.

(2) Much of the data supplied will have been derived by experiment. Consequently, due to inevitable errors, this data may be only approximate, and, as a result, the plotted values may not lie exactly on either a straight line or a smooth curve. In such cases, the procedure is to draw the line which most nearly fits the plotted points, the errors on the one side, due to displaced points, being approximately equal to those on the other.

(3) There should be as little writing as possible on a graph. The title and scales should be placed so as not to interfere with its use. The axes should be named and the intervals neatly numbered along the margins. Unless the origin is of necessity located within the ruled page, no writing should appear in the body of the graph.

EXERCISES.

1. In an experiment with a screw jack, the following results were obtained, F being the force applied to raise a load R. Illustrate graphically.

F lb.	0·8	1·3	1·8	2·3	2·8	3·3	3·8	4·3
R lb. ..	12·5	25·0	38·5	51·0	64·5	78·0	91·0	103·5

Find the force F when (i) $R = 56$ lb., (ii) $R = 112$ lb.

2. As the armature of an alternator rotates, the current delivered at various angles of rotation is given in the following table. Draw a graph to show the variation in current during each revolution of the armature.

Angle	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
Amps.	0	5·00	8·66	10	8·66	5·00	0	−5·00	−8·66	−10	−8·66	−5·00	0

What type of curve is this ?

3. From the following data, draw a graph to show the relationship of temperature to height above sea level.

Height above sea level..	0	5000	10000	15000	20000	25000	30000	35000	40000	45000
Temperature	15°C.	5° C.	−5° C.	−15° C.	−25° C.	−35° C.	−45° C.	−55° C.	−57° C.	−57° C.

What are the temperatures at 31,000 feet and 17,000 feet ?

4. Draw a graph to show the lift co-efficients (factor representing the lift of an aerofoil) for the angles of attack (the angle between an aerofoil chord and the direction of the relative air flow) given in the following table :—

Angle of Attack ..	−4°	−2°	0°	2°	4°	6°	8°	10°	12°	14°	15°	16°	18°
Lift Co-efficient	−0·14	+0·02	0·14	0·32	0·46	0·60	0·76	0·90	1·04	1·18	1·22	1·16	1·02

Exercises—*continued.*

5. Draw a further graph to show the effect of the addition of a slot to the same aerofoil.

Angle of Attack ..	0°	2°	4°	8°	12°	16°	20°	24°	26°	28°	30°
Lift Co-efficient ..	0·04	0·16	0·34	0·56	0·90	1·24	1·50	1·70	1·76	1·76	1·64

6. Given that 5 kilograms = 11 lb., draw a graph for reading off the number of kilograms corresponding to any number of lb. up to 28 lb.

7. The mass of 1 c. ft. of water in lb., for various temperatures, is given in the following table. Plot a curve showing how the mass of 1 c. ft. of water varies with the temperature. At what temperature is the mass greatest ? What is the mass at 180°F. ?

Temperature F. ..	32°	39°	46°	80°	100°	120°	140°	160°	200°
Mass of 1 c. ft. of water, in lb.	62·42	62·425	62·42	62·23	62·02	61·71	61·38	61·00	60·08

PLANE FIGURES.

(Revise all definitions in the Geometry Section of Preliminary Course.)

1. TRIANGLES.

(i) A **triangle** is a closed plane figure bounded by three straight lines, e.g., Fig. 1.

(ii) A right-angled triangle is a triangle with one of its angles equal to a right angle, e.g., Fig. 2.

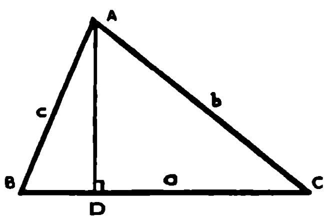


Fig. 1.

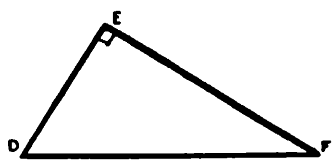


Fig. 2.

Note.—The sides of a triangle are named by small letters corresponding to the names of the angles they are opposite, e.g., side a is opposite angle A and so on.

(iii) **Formulae for Area.**

Area of a triangle $= \frac{1}{2}$ base $\times$ perpendicular height.

i.e., $= \frac{1}{2}\ BC \times AD.$

In the right-angled $\triangle DEF$, if we take ED as the base, FE will be the perpendicular height, or vice versa.

$\therefore$ Area of $\triangle DEF = \frac{1}{2}\ DE \times EF.$

2. QUADRILATERALS.

(i) A **Quadrilateral** is a closed plane figure bounded by four straight lines, e.g., Fig. 3.

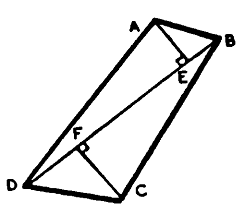


Fig. 3.

2. **Quadrilaterals**—*continued.*

(ii) **Formula for Area.**

The area of the quadrilateral may be found by dividing the figure into two triangles by the diagonal BD.

$$\begin{aligned} \text{Area of quad. } ABCD &= \text{area of } \triangle ADB + \text{area of } \triangle CDB \\ &= \tfrac{1}{2}\, BD \times AE + \tfrac{1}{2}\, BD \times CF \\ &= \tfrac{1}{2}\, BD\,(AE + CF) \end{aligned}$$

(AE and CF are termed **off-sets.**)

Thus area $= \frac{1}{2}$ diagonal $\times$ sum of off-sets.

3. **PARALLELOGRAM.**

(i) A parallelogram is a quadrilateral whose opposite sides are parallel, e.g., Fig. 4. It is a property of this figure that opposite sides are equal.

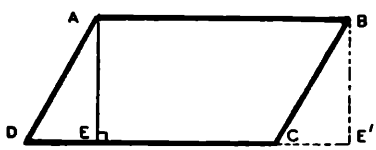


Fig. 4.

(ii) Area of parallelogram $ABCD$ = twice area of $\triangle ADC$.

Therefore from formula for area of triangle—

Area of parallelogram $ABCD = DC \times AE$ = base $\times$ perp. height.

4. **RECTANGLE.**

(i) A rectangle is a parallelogram, one of whose angles is a right angle, e.g., Fig. 5. This necessitates all the angles being right angles.

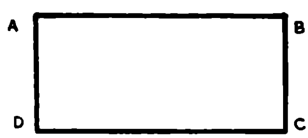


Fig. 5.

(ii) Area of rectangle $ABCD = AD \times DC$.

5. TRAPEZIUM.

(i) A trapezium is a quadrilateral having one pair of opposite sides parallel, e.g., Fig. 6.

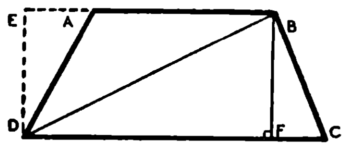


Fig. 6.

(ii) Area of trapezium $ABCD =$ Area of triangle ABD + area of triangle BDC.

$= (\frac{1}{2} AB \times DE) + (\frac{1}{2} DC \times BF)$

But $BF = DE$

$\therefore$ Area $= \frac{1}{2} (AB + DC) DE$

i.e., Area $= \frac{1}{2}$ sum of parallel sides $\times$ perpendicular distance between them.

6. IRREGULAR PLANE FIGURE.

An irregular plane figure is a closed plane figure bounded by straight lines of various lengths.

The area is the sum of the areas of the triangles into which the figure may be divided.

EXAMPLES.

(1) Find the area of a triangle of base $10\frac{1}{2}$ in. and perpendicular height (by scaling) $7\frac{1}{4}$ in.

Area $= \frac{1}{2}$ base $\times$ perp. height.

$= \frac{1}{2} \times 10\frac{1}{2} \times 7\frac{1}{4}$ sq. in.

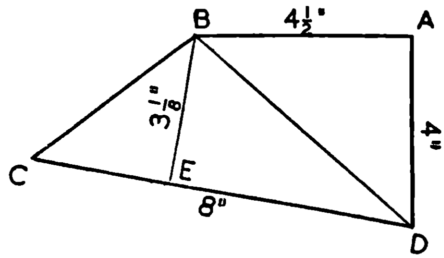
$= \frac{1 \times 21 \times 29}{2 \times 2 \times 4}$ sq. in.

$= \frac{609}{16}$ sq. in.

$= 38\frac{1}{16}$ sq. in.

Examples—*continued.*

(2) Calculate the area of the given figure, where $\angle BAD$ is a right-angle and BE is perpendicular to CD.



$$\begin{aligned}
\text{Area of } \triangle ABD &= \tfrac{1}{2} \times 4 \times 4\tfrac{1}{2} \text{ sq. in.} \\
&= 9 \text{ sq. in.} \\
\text{Area of } \triangle BCD &= \tfrac{1}{2} \times 8 \times 3\tfrac{1}{8} \text{ sq. in.} \\
&= 12{\cdot}5 \text{ sq. in.} \\
\text{Total area} &= \mathbf{21{\cdot}5 \text{ sq. in.}}
\end{aligned}$$

EXERCISES.

Note.—Draw figures always. Mark given dimensions on them. Give answers correct to second decimal place.

1. A ditch, with straight sides, is 5 feet deep, 4 ft. 7 in. wide at top, and 3 ft. 11 in. wide at the bottom. What is its area of cross section ?

2. Find the area of a triangular field, having base of 250 yards and perpendicular height of 125 yards.

3. Find the area of a rectangular piece of metal of sides 7 and 5 cm.

4. Find the area of the quadrilateral $ABCD$ from the following measurements :—

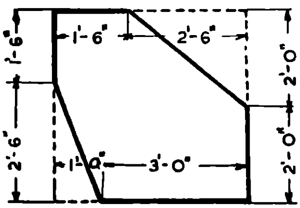
$$BD = 7 \text{ cm}$$
$$\left.\begin{aligned} AE &= 4 \text{ cm} \\ CF &= 5 \text{ cm} \end{aligned}\right\} \text{Where } AE \text{ and } CF \text{ are perpendicular to } BD.$$

5. Find the area of a parallelogram $ABCD$ such that $AB = 10$ cm
$AE = 15$ cm
where AE is perpendicular to CD.

6. The gable end of a building is 30 feet wide ; the height from the ground to each gutter is 22 feet, and from the ground to the ridge tile is 30 feet. Find the area of the end.

Exercises—*continued.*

7. The figure represents a gusset plate of mild steel weighing 25·5 lb. per square foot. Find its weight.



8. An unfenced square field has an area of $6\frac{1}{5}$ acres. Find, to the nearest 10 yds., the side of the field.

PLANE FIGURES—*continued.*

1. CIRCLE.

(i) A circle is the area bounded by the path traced out by a point moving so that it is at a constant distance from another fixed point. The constant distance is termed the radius, the fixed point the centre.

(ii) **Note.**—The ratio of the circumference to the diameter is the same for all circles, namely 3·1416 correct to 5 significant figures. This value is denoted by the symbol π. ($3\frac{1}{7}$ or $\frac{22}{7}$ is an approximation of the value of π, correct to 3 significant figures.)

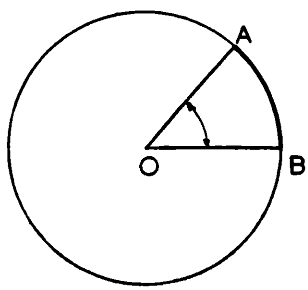
$$\frac{\text{Circumference}}{\text{Diameter}} = \pi$$

$$\text{Circumference} = \pi d \text{ (or } 2\pi r)$$

where d = diameter of circle

r = radius of circle.

(iii) An **arc** is part of the circumference. Let the arc AB subtend an angle AOB at the centre O.



Then **arc** $AB = \dfrac{\angle AOB}{360°} \times$ circumference.

$$= \frac{\angle AOB}{360°} \times \pi d.$$

where d = diameter.

(iv) **Area of circle** $= \pi r^2$.

where r = radius

and $\pi = 3{\cdot}1416$ or $3\frac{1}{7}$ (approx.)

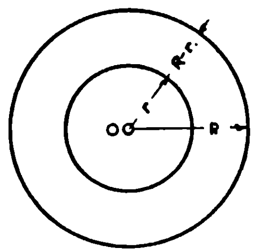
or **area** $= \mathbf{0{\cdot}7854}d^2$ i.e., $\pi \left(\dfrac{d}{2}\right)^2 = \dfrac{\pi}{4} d^2$

where d = diameter.

2. ANNULUS.

(i) Two circles are concentric when they are described about the same centre.

An annulus is the area bounded by the circumferences of the two concentric circles.



(ii) **Area of annulus** = Area of outer circle − area of inner circle.

$$= \pi R^2 - \pi r^2$$
$$= \pi (R^2 - r^2)$$
$$= \pi (R + r)(R - r)$$

$= \pi \times$ **sum of radii** $\times$ **difference of radii.**

EXAMPLES.

(1) A hollow propeller shaft has an external diameter of 18 inches and an internal diameter of 9 in. Find the area of cross section.

$$\textbf{Area} = \pi \times (9 + 4{\cdot}5) \times (9 - 4{\cdot}5) \text{ sq. in.}$$
$$= \frac{22}{7} \times \frac{27}{2} \times \frac{9}{2} \text{ sq. in.}$$
$$= \textbf{191 sq. in. (approx.).}$$

(2) A circular strut has $3\frac{1}{8}$ in. outside diameter, and two radial rivet holes are drilled on the same cross section, their centre lines making an angle of 60°. Find the shorter distance between the rivet centres measured around the circumference.

$$\text{Arc} = \frac{60}{360} \times \pi \times \text{diameter}$$
$$= \frac{\overset{1}{\cancel{60}}}{\underset{\underset{3}{\cancel{6}}}{\cancel{360}}} \times \frac{\overset{11}{\cancel{22}}}{7} \times \frac{25}{8} \text{ in.}$$
$$= \frac{275}{168} \text{ in.}$$
$$= \textbf{1·64 in.}$$

EXERCISES.

Note.—(i) Answers correct to 2 decimal places.

(ii) Use $\pi = \frac{22}{7}$ unless otherwise stated.

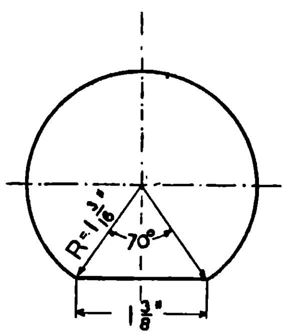
(iii) Draw a figure whenever problem refers to one.

1. A boiler is 7′ in diameter, and the front end plate is pierced for a furnace tube by a hole 2′ 4″ in diameter. The plate is made of steel weighing 15·3 lb. per sq. foot. What is the weight of the front end plate ?

2. Find the area of a circular cover, 3 feet in diameter, which has 9 holes bored through it, each 1 inch in diameter.

3. The diameter of a lever safety valve is 3 inches, and steam blows off at 90 lb. per sq. in. Find the total pressure on the valve when it is about to blow off.

4. A uniform tube of sheet aluminium has the cross-section illustrated.



(i) Find, in sq. feet, the area of sheet required to make 1 foot of tube.

(ii) If the sheet aluminium used weighs 14 oz. per sq. foot, what would be the weight of 7 feet of tubing ?

5. The International Aircraft markings for British aircraft are in the form of concentric circles, red, white and blue from the centre outwards. The distances of the outside edge of each colour from the centre are 3·5 in., 10·5 in., 17·5 in. respectively. Find the areas covered by each colour separately.

6. The area of a circle is 17·4 sq. cm. Find its diameter.

REVISION.

1. Revise Lectures 5 to 10.

2. Commit the following rules to memory :—

Figure.	Area.
1. Triangle	$\frac{1}{2}$ base $\times$ perpendicular height.
2. Quadrilateral	$\frac{1}{2}$ diagonal $\times$ sum of offsets.
3. Parallelogram ..	Base $\times$ perpendicular height.
4. Rectangle	Length $\times$ breadth.
5. Square	Square of length of one side.
6. Trapezium	$\frac{1}{2}$ sum of parallel sides $\times$ perpendicular distance between the parallel sides.
7. Irregular Linear Figure	Sum of areas of the triangles into which it can be divided.
8. Circle	πr^2 or $\dfrac{\pi d^2}{4}$
9. Annulus	π $\times$ sum of radii $\times$ difference of radii.

Also :—

Circumference of Circle $= \pi d$.

Arc subtending Angle $A = \dfrac{A}{360} \times \pi d$.

$$\pi = \frac{22}{7} \text{ (approx.).}$$

REVISION EXERCISES.

1. What is the ratio of 1 yard to 1 cm. ?

2. Given that the acceleration produced by a force is directly proportional to the force, find the acceleration produced by a force of 10 tons, if a force of 6 tons gives an acceleration of 4 ft. per sec. per sec.

3. The heat received from a given lamp is inversely proportional to the square of the distance from the lamp. If 14 units of heat are received at a distance of 9 inches, find the heat received at 7 inches distance.

Revision Exercises—*continued.*

4. The following are wing spans of various aircraft: 32 ft. 3 in., 40 ft. 0 in., 36 ft. 10 in., 39 ft. 6 in., 56 ft. 4 in., 50 ft. 0 in., 84 ft. 0 in., 69 ft. 2 in.; find the average span.

5. What percentage of the above total is the 56 ft. 4 in. span?

6. Evaluate :—

(*a*) $\sqrt{17 \cdot 6}$;

(*b*) $\sqrt{0 \cdot 00139}$;

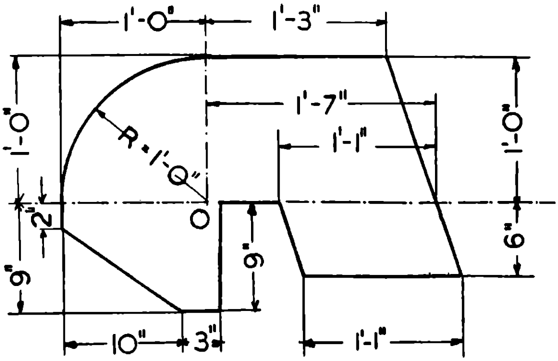
(*c*) $\sqrt{134 \cdot 271}$.

7. The following 2-hourly readings of electric current in amperes were recorded on an ammeter for a continuous 24-hour period :—

3 (0000 hr.), 4, 10, 31, 38, 6, 25 (1200 hr.), 48, 8, 19, 29, 2, 5 (2400 hr.).

Construct a suitable graph to show the variation in current during the day. What amperage was being used at 1515 hours?

8. The figure represents a gusset plate of aluminium alloy weighing 15 oz. per sq. foot. Find its weight.



PRELIMINARY COURSE

ALL CADETS

Algebra 1B/1

SYMBOLIC EXPRESSION, SUBSTITUTION, POSITIVE AND NEGATIVE QUANTITIES.

1. SYMBOLIC EXPRESSION.

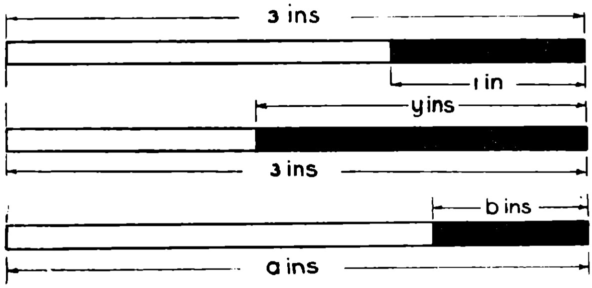


FIG. I

FIG. II

FIG. III

In Algebra, letters are used to represent Numbers.

Let Figures I. and II. represent a rod 3 inches long.

(i) In Fig. I. a portion of length 1 inch is painted black. The rest is white. The portion painted white is, therefore, 3 inches — 1 inch = 2 inches.

(ii) In Fig. II. the portion painted black is y inches. The rest is white. The portion painted white is then 3 inches — y inches or $(3 - y)$ inches.

(iii) In Fig. III. the length of the rod is a inches and the portion painted black is b inches. Hence the portion painted white = a inches — b inches = $(a - b)$ inches.

The distinction between the arithmetical statement of (i), i.e., the length of the white portion = $(3 - 1)$ inches = 2 inches, and the algebraic statement of (iii), i.e., the length of the white portion = $(a - b)$ inches should be noted. The arithmetical statement refers to one specific rod, the length of which is 3 inches and of which the portion painted black is 1 inch, whereas the algebraic statement generalizes and refers to any rod, portion of which is painted black and the rest white. a and b can be any lengths we like, but the METHOD for determining the length of the white portion is to subtract b inches from a inches.

1. **Symbolic Expression**—*continued.*

The signs, $+$, $-$, $\times$ and $\div$ have the same meaning in Algebra as in Arithmetic. The chief difference, however, between Algebra and Arithmetic lies in the fact that in the former we use letters or symbols whilst in the latter we use numbers only. To the letters or symbols we may give any value we please.

Thus, 2×3 is always equal to 6, but $2 \times a$ will have different values according to the value of the symbol a. When $a = 3$, $2 \times a = 6$; when $a = 5$, $2 \times a = 2 \times 5 = 10$.

$2 \times a$ is more quickly and conveniently written $2a$.

Similarly, $a \times \frac{1}{2} = \frac{1}{2}a$ or $\frac{a}{2}$.

Also, just as $5 \times 9 = 9 \times 5$, so $a \times b = b \times a$, and the shortened form is ab or ba.

When two symbols, or a number and a symbol, are written together without any sign between them, it always means that they are multiplied together, thus $5b$ means 5 times b; ac means a times c; $2xy$ means 2 times x times y.

Symbols may also be used to denote quantities (money, weight, &c.), as well as plain numbers, and these may be changed from one unit to another in exactly the same way as in Arithmetic.

Thus :-

£5 = (20×5) shillings or (240×5) pence.

£a = $20a$ shillings = $240a$ pence.

132 ft. = $\frac{132}{3}$ yds. or $\frac{132}{66}$ chains.

a ft. = $\frac{a}{3}$ yds. or $\frac{a}{66}$ chains.

For conciseness we can write $2 \times 2 \times 2 = 2^3$, $3 \times 3 = 3^2$, $5 \times 5 \times 5 \times 5 = 5^4$, &c. In the same way we can write—

$a \times a = a^2$

$b \times b \times b = b^3$

$a \times a \times b \times b \times b = a^2 \times b^3 = a^2b^3.$

2. SUBSTITUTION.

If we know the numerical values of all symbols we may substitute the numbers in place of the symbols and so find the numerical value of a whole expression.

Thus, if $a = 3$, then
$$\begin{aligned}
2a &= 2 \times 3. \\
&= 6. \\
a^2 &= a \times a. \\
&= 3 \times 3. \\
&= 9. \\
4a^3 &= 4 \times a \times a \times a, \\
&= 4 \times 3 \times 3 \times 3, \\
&= 108.
\end{aligned}$$

Again, if
$$\begin{aligned}
a &= 2, \\
b &= 3, \\
c &= 4, \\
\text{then } abc &= 2 \times 3 \times 4, \\
&= 24. \\
a^2b &= 2 \times 2 \times 3, \\
&= 12. \\
2ab^2c^3 &= 2 \times 2 \times 3 \times 3 \times 4 \times 4 \times 4, \\
&= 2304.
\end{aligned}$$

Again, if
$$\begin{aligned}
a &= 0, \\
b &= 1, \\
c &= 3, \\
\text{then } a^2 &= 0 \times 0 = 0, \\
abc &= 0 \times 1 \times 3, \\
&= 0. \\
b^4 &= 1 \times 1 \times 1 \times 1, \\
&= 1. \\
b^c &= 1^3, \\
&= 1 \times 1 \times 1, \\
&= 1.
\end{aligned}$$

NOTE CAREFULLY.

1. Wherever 0 is a factor of an expression, that expression must be equal to 0, since 0 times any factor always equals 0.

2. Where little a, b, c, x, y, &c. are given, always write them in small letters; do not use the capitals A, B, C, X, Y, &c., unless the capital letters are given.

3. The symbol $\times$ is the multiplication sign. Confusion is likely to occur if a distinction is not made between it and the algebraical symbol x, which should always be written as just shown.

2. Substitution—*continued.*

4. In setting out your work in all exercises in Algebra, note the following points :—

(*a*) Use the body of the page for working and statements.

(*b*) Always put the final answer clearly defined by itself and clearly marked at the foot of each example.

EXAMPLES.

(1) If $x = 3$, $y = 5$, $c = 8$, find the value of $cy - 10x$.

$$\begin{aligned} cy - 10x &= 8 \times 5 - 10 \times 3, \\ &= 40 - 30, \\ &= \mathbf{10}. \qquad \textbf{Answer.} \end{aligned}$$

(2) If $p = 4$, $q = 9$, $r = 1$, find the value of $\dfrac{q + r}{p + 1}$.

$$\frac{q + r}{p + 1} = \frac{9 + 1}{4 + 1} = \frac{10}{5} = \mathbf{2}. \qquad \textbf{Answer.}$$

EXERCISES.

1. Express £x in half-sovereigns; in half-crowns; in sixpences.

2. Express a miles in chains; in yards; in feet.

3. If $a = 5$, $b = 3$, $c = 1$, $x = 4$, $y = 0$, find the value of :—

(*a*) $2ab^2c^3x$.

(*b*) $9b^3x^2y$.

(*c*) $\frac{4}{25} a^2cy^2$.

(*d*) $\frac{6}{125} xy^3$.

(*e*) $\frac{1}{2}b^3c^3x^2$.

(*f*) $a^2 + b^2 + c^2 - x^2 - y^2$.

(*g*) $\dfrac{x^2}{25} - \dfrac{c^2}{81} - \dfrac{bc}{9} + \dfrac{bcx}{36}$.

4. Prove that the following statements are true when $x = 1$, $y = 2$, $z = 4$:—

(*a*) $x^2 + y^2 + z^2 = 21$.

(*b*) $x^2y + y^2z = 18$.

(*c*) $\dfrac{x}{y} + \dfrac{y}{z} + \dfrac{z}{x} = 5$.

(*d*) $\dfrac{z^2}{y} - \dfrac{y^2}{x} + \dfrac{x^2}{z} = 4\frac{1}{4}$.

3. POSITIVE AND NEGATIVE QUANTITIES.

(i) KINDS OF QUANTITIES.

As a result of observation and experience, we know that many **physical** quantities exist in two directly opposite states. These **states** are expressed by such words as :—

GAIN	and its opposite	LOSS
GOING	„	COMING
RISE		FALL
APPROACHING		RECEDING
CLOCKWISE		ANTI-CLOCKWISE
BEFORE		AFTER

It must be remembered that the "**opposite**" is always the condition or quantity opposed to the one being considered, e.g., if an object is **COMING** towards the observer then the opposite of this would be if it was **GOING** away from the observer.

We distinguish these two states by calling one **POSITIVE** and the opposite one **NEGATIVE,** e.g., if clockwise rotation (i.e., rotation in same direction as the hands of a clock) is called positive then anti-clockwise rotation is called negative and vice versa

(ii) + AND − SIGNS.

The + and − signs have already been used to denote the **OPERATIONS** of **Addition** and **Subtraction** respectively. That is, **whatever kind** of quantity is being considered, + 4 will always stand for what **increases** that quantity by 4 units, and − 4 will stand for what **decreases** that quantity by 4 units.

Consider an aircraft in flight :—

Then (1), if the aircraft is **RISING,** + 100 feet could be taken to stand for what increases the total rise, i.e., **+ 100 feet stands for a RISE of 100 feet ;** so also − 100 feet would stand for what decreases the total rise, i.e., **− 100 feet will stand for a FALL of 100 feet,** which is 100 feet in the opposite direction to a rise.

Also (2), if the aircraft is **FALLING,** + 100 feet could be taken to stand for what increases the total fall, i.e., **+ 100 feet stands for a FALL of 100 feet ;** so also − 100 feet would stand for what decreases the total fall. i.e., **− 100 feet will stand for a RISE of 100 feet,** which is 100 feet in the opposite direction to a fall.

Hence, it can be seen that the **+ and − signs can be used as marks or symbols to distinguish between quantities of directly opposite kinds.** Such quantities have been named above as positive and negative quantities.

(iii) POSITIVE AND NEGATIVE QUANTITIES.

A quantity to which the $+$ sign is prefixed is called a **POSITIVE QUANTITY** and a quantity to which the $-$ sign is prefixed is called a **NEGATIVE QUANTITY.** When neither $+$ or $-$ sign is prefixed, the $+$ sign is to be understood.

The $+$ is the **Positive Sign.**
$-$ is the **Negative Sign.**

In Example (1) above, a **Rise** was taken as the positive quantity and a **Fall** was a negative quantity.

In Example (2), a **Fall** was taken as the positive quantity and hence a **Rise** was a negative quantity.

Note.—If either one of two opposed quantities is chosen as positive then the other will be negative. The choice is **quite arbitrary** but once it has been made, it **must be adhered to** throughout the calculations concerned.

Note.—In dealing with a particular magnitude a—
Positive quantity always indicates an **Increase.**
Negative quantity always indicates a **Decrease.**

(iv) **THE ABSOLUTE MAGNITUDE** of a quantity is its magnitude considered independently of its sign. Thus a 100 ft. rise and 100 ft. fall are equal in absolute magnitude but opposite in sign.

i.e., $+ 100$ and $- 100$ are equal in absolute magnitude but the first is positive and the second negative.

To indicate that the absolute magnitude is being considered, the symbol is placed between vertical bars.

e.g., $+ a$ and $- a$ are two opposed quantities. The absolute magnitude is $|a|$ in both cases.

EXERCISES.

1. What is the net property of a man who—
   (i) has £50 and owes £49;
   (ii) has £50 and owes £51;
   (iii) has £3 and owes £20.
2. A liquid at a temperature $+ 20°$ C. is cooled through $30°$ C. What is its final temperature?
3. Alcohol freezes at $- 112°$ C. and boils at $78°$ C. Through how many degrees must alcohol be heated from its frozen state before it boils?
4. A transport van moved 20 miles North, then 10 miles South, then 14 miles North and finally 4 miles South. How far North of its starting point is its final position? (Take North as the positive direction and express the answer as a positive or negative answer.) What would the answer have been if the respective values had been 20 miles South, 10 miles South, 14 miles North, 4 miles North?
5. Find the value of $a - b + c$ and of $-a + b - c$, when $a = 1$; $b = -2$ and $c = 3$.
6. Find the value of $-a + b - c$, when $a = 1$, $b = -2$, $c = -1$ and also when $a = -2$, $b = -1$, $c = -3$.

ALGEBRAIC PROCESSES—ADDITION, BRACKETS, SUBTRACTION.

1. ADDITION.

The important rule to remember in Addition is that only **like terms** (i.e., those containing exactly the same symbols) can be added together into a single term. **Unlike terms** (those in which the symbols differ in any way) cannot be added together to form a single term.

Consider :—

(i) $+ 10a + 6a = + 16a.$
(ii) $+ 10a - 6a = + 4a.$
(iii) $- 10a + 6a = - 4a.$
(iv) $- 10a - 6a = - 16a.$

In Examples (i) and (iv) we see that the answer is obtained by adding the two numbers and taking the sign which precedes them both ; and in Examples (ii) and (iii) we see that the answer is obtained by subtracting the two numbers and taking the sign which precedes the greater of them. The addition and subtraction is done only with the co-efficients, i.e., the numbers which stand in front of the symbols.

Thus we may state **THE TWO FUNDAMENTAL RULES OF ADDITION AS FOLLOWS :**

RULE 1.—When the signs are the same, add the co-efficients and take the same sign.

RULE 2.—When the signs are different, subtract the co-efficients and take the sign of the greater.

When the expressions to be added contain two or more terms, the work can be done most simply by arranging them one under the other as in Arithmetic. Care must be taken, however, that only like terms appear in the same vertical column ; never put unlike terms in the same column. Each column of like terms is then added according to the above rules.

2. BRACKETS.

In Arithmetic, when any numbers are placed within brackets, it means that these numbers must be considered as a group together and should really be worked out first. Thus $8 + (7 + 5)$ means that we first add 7 and 5 and then add this answer to 8.

In Algebra, however, this cannot always be done, since the symbols inside the brackets may be different, in which case they cannot be added or subtracted into a single term. It is, therefore, necessary to see whether we can remove the brackets before adding or subtracting. Consider four simple examples :

Example (i) : $9 + (5 + 2)$.

This means add 5 and 2, and then add the result to 9. If we begin by adding 5 on to 9 we have not added enough, but must add two more, i.e., $9 + (5 + 2) = 9 + 5 + 2$.

2. **Brackets**—*continued.*

Example (ii): $9 + (5 - 2)$.

This means take 2 from 5 and add the result to 9. If we begin by adding 5 on to 9 we have added too much and so must take 2 away, i.e., $9 + (5 - 2) = 9 + 5 - 2$.

Example (iii): $9 - (5 + 2)$.

This means add 5 and 2 and take the result from 9. If we begin by taking 5 from 9 we have not taken away enough, but must take away 2 more, i.e., $9 - (5 + 2) = 9 - 5 - 2$.

Example (iv): $9 - (5 - 2)$.

This means take 2 from 5 and take the result from 9. If we begin by taking 5 from 9 we have taken away too much, and so must add on 2, i.e., $9 - (5 - 2) = 9 - 5 + 2$.

From Examples (i) and (ii) we see that when a + sign preceded the bracket and the bracket was removed, no change occurred in the numbers inside the bracket. From Examples (iii) and (iv), however, we see that when a − sign preceded the bracket and the bracket was removed, the signs of the numbers inside the bracket were changed, i.e., in Example (iii), $-(5 + 2)$ became $-5 - 2$ and in Example (iv), $-(5 - 2)$ became $-5 + 2$.

Thus we have established the two rules for removing brackets which are given below. These are very important, and must be learnt thoroughly.

> **RULE 1.**—When a bracket which is preceded by a + sign is removed, the signs inside the bracket remain unchanged.
>
> **RULE 2.**—When a bracket which is preceded by a − sign is removed, all the signs inside the bracket must be changed.

Further illustrations of these rules as applied to Algebra are given below, and should be studied carefully.

$$\begin{array}{ll}
a + (b + c) = a + b + c & \text{Rule (1)} \\
a - (b + c) = a - b - c & \text{,, (2)} \\
2a + (3b - c) = 2a + 3b - c & \text{,, (1)} \\
3a - (b - 2c) = 3a - b + 2c & \text{,, (2)} \\
a + (-2b - c) = a - 2b - c & \text{,, (1)} \\
5a - (-3b + c) = 5a + 3b - c & \text{,, (2)}
\end{array}$$

When a number or symbol stands immediately in front of a bracket with no sign between them, it means that everything inside the bracket must be multiplied by this number or symbol when the bracket is removed. The rules of signs already given also apply.

Thus—

$$\begin{array}{l}
2\,(a + b) = 2a + 2b \\
-\,3\,(2a - 3b) = -\,6a + 9b \\
a\,(b - c) = ab - ac \\
-\,x\,(a + 2b) = -\,ax \quad 2bx \\
b\,(3b + c) = 3b^2 + bc \\
-\,2x\,(a - x) = -\,2ax + 2x^2.
\end{array}$$

2. Brackets—*continued.*

Similarly, when a fraction has more than one term in the numerator, the numerator must be considered as being in a bracket.

Thus—

$$\frac{x + 4}{2} = \tfrac{1}{2}\,(x + 4)$$

This must be remembered, especially when a $-$ sign precedes the fraction, since it affects the signs of the terms in the numerator, thus :—

$$-\frac{2x + 6}{2} = -\tfrac{1}{2}\,(2x + 6) = -x - 3.$$

$$-\frac{6 - 9}{3} = -2 + 3.$$

When an expression contains two or more brackets, each may be removed separately, and then any like terms (i.e., those having the same letters) may be collected together and added into a single term. Examples of these steps are given below :—

EXAMPLES.

(1) $6a - (4a - 2a)$

$$\begin{aligned} &= 6a - 4a + 2a, \\ &= 6a + 2a - 4a, \\ &= 8a - 4a, \\ &= 4a. \end{aligned}$$

(2) $5ab - (2ab + 3ab)$

$$\begin{aligned} &= 5ab - 2ab - 3ab, \\ &= 5ab - 5ab, \\ &= 0. \end{aligned}$$

(3) $(x - 2) - (3 + 4x) + (6x + 1)$

$$\begin{aligned} &= x - 2 - 3 - 4x + 6x + 1, \\ &= x - 4x + 6x - 2 - 3 + 1, \\ &= 3x - 4. \end{aligned}$$

(4) $2\,(2a - b) - 3\,(a - 2b)$

$$\begin{aligned} &= 4a - 2b - 3a + 6b, \\ &= 4a - 3a - 2b + 6b, \\ &= a + 4b. \end{aligned}$$

(5) $\dfrac{3x - 9}{3} + \dfrac{4x - 12}{2} - \dfrac{8x + 12}{4}$

$$\begin{aligned} &= (x - 3) + (2x - 6) - (2x + 3), \\ &= x - 3 + 2x - 6 - 2x - 3, \\ &= x + 2x - 2x - 3 - 6 - 3, \\ &= x - 12. \end{aligned}$$

Examples—*continued.*

(6) Add $3x^2 - 5x$ and $2x - 3$.

$$\begin{array}{rrr} 3x^2 & -\ 5x & \\ & +\ 2x & -\ 3 \\ \hline 3x^2 & -\ 3x & -\ 3 \\ \hline \end{array}$$ **Answer.**

(7) Add $2a + 3b - 4c$, $3a - 2b + 4c$ and $a + 5b + 6c$.

$$\begin{array}{rrr} 2a & +\ 3b & -\ 4c \\ 3a & -\ 2b & +\ 4c \\ a & +\ 5b & +\ 6c \\ \hline 6a & +\ 6b & +\ 6c \\ \hline \end{array}$$ **Answer.**

(8) Add $3\ (a - b + c)$ and $4\ (a - b - c)$.

First remove brackets.

$$\begin{array}{rrr} 3a & -\ 3b & +\ 3c \\ 4a & -\ 4b & -\ 4c \\ \hline 7a & -\ 7b & -\ c \\ \hline \end{array}$$ **Answer.**

(9) Add $4x - 6x^2 - 1 + 2x^3$, $3x^2 - 4 - x^3 + 5x$, and $12 - x$.

In cases like this, each expression should be re-arranged with the highest power at the beginning and the others in descending order.

$$\begin{array}{rrrr} 2x^3 & -\ 6x^2 & +\ 4x & -\ 1 \\ -\ x^3 & +\ 3x^2 & +\ 5x & -\ 4 \\ & & -\ x & +\ 12 \\ \hline x^3 & -\ 3x^2 & +\ 8x & +\ 7 \\ \hline \end{array}$$ **Answer.**

(10) Prove the following by removing the brackets :—

$$2\ (3x + 12) + 3\ (x - 4) - 4\ (2x + 3) = x.$$

$$\begin{aligned} &2\ (3x + 12) + 3\ (x - 4) - 4\ (2x + 3) \\ &= 6x + 24 + 3x - 12 - 8x - 12 \\ &= x + 24 - 24 = x. \end{aligned}$$

EXERCISES.

Find sum of :—

(1) $3ab - 7ab + ab - 2ab + 9ab$.

(2) $11x^2y - 8x^2y - 2x^2y + 4x^2y - x^2y$.

(3) $-\ \frac{5}{8}a^2 + \frac{2}{3}a^2 - a^2 - 2a^2$.

(4) $a^2 - b^2 + c^2$, $-\ a^2 - b^2 - c^2$, $a^2 + b^2 + c^2$.

(5) $2a + 3b - 4c$, $3a - 2b + 4c$, $a + 5b + 6c$.

(6) $4ax - 3by + 5cz$, $7ax + 8by - 2cz$, $2ax - 2by + cz$.

Exercises—*continued.*

(7) $3a^3 - 2c^3 - d^3$, $b^3 + c^3 + 4d^3$, $a^3 - 3b^3 - 4c^3$.

(8) $7x^2yz - 5xyz^2$, $3xy^2z - 4x^2yz$, $-5xy^2z - 7xyz^2$, $2x^2yz - 4xy^2z + 6xyz^2$.

(9) $4\,(a + b + c)$, $3\,(2a - b - c)$, $8\,(b - a + 2c)$.

(10) $\frac{2}{3}a + \frac{1}{3}b$, $\frac{1}{3}a - c$, $\frac{5}{3}b + 6c$.

(11) $\frac{3}{4}\,(8x - 12y)$, $\frac{2}{3}\,(6x - 9y)$, $\frac{1}{6}\,(12x + 30y)$.

Prove the following by removing the brackets :—

(12) $6 + (x - 2) - (3 + 4x) + (6x + 1) = 3x + 2$.

(13) $(3x - 2) - (4x - 5) + (x + 7) = 10$.

(14) $(a + b - c) - (a - b - c) + (a - b + c) = a + b + c$

(15) $3\,(a - b + c) - 4\,(b + a - c) - 2\,(c - a - b) = a - 5b + 5c$

(16) $\dfrac{3x - 9}{3} + \dfrac{4x - 12}{2} - \dfrac{8x + 12}{4} = x - 12.$

(17) $\dfrac{6x - 8}{2} + \dfrac{10x - 5}{5} - \dfrac{14x - 21}{7} = 3x - 2.$

(18) $\dfrac{8 - 9x}{3} - \dfrac{7 - 21x}{7} + \dfrac{20 + 25x}{5} = 5x + 5\frac{2}{3}.$

3. SUBTRACTION.

$2a$ subtracted from $5a = 5a - 2a = 3a$.

$2a$,, ,, ,, $-5a = -5a - 2a = -7a$.

$-2a$,, ,, ,, $5a = 5a - (-2a) = 5a + 2a = 7a$.

$-2a$,, ,, ,, $-5a = -5a - (-2a) = -5a + 2a = -3a$.

From these examples it will be seen that, with single terms, subtraction differs very little from addition.

With expressions containing two or more terms, subtraction may also be done by using brackets, thus :—

Example (1) :

$2a - 3b$ subtracted from $5a + 2b$

$$\begin{aligned}
&= 5a + 2b - (2a - 3b) \\
&= 5a + 2b - 2a + 3b \\
&= 5a - 2a + 2b + 3b \\
&= 3a + 5b.
\end{aligned}$$

Example (2) :

$5x^2 - 3x + 2$ subtracted from $7x^2 - 5x + 6$

$$\begin{aligned}
&= 7x^2 - 5x + 6 - (5x^2 - 3x + 2) \\
&= 7x^2 - 5x + 6 \quad 5x^2 + 3x - 2 \\
&= 7x^2 - 5x^2 - 5x + 3x + 6 - 2 \\
&= 2x^2 - 2x + 4.
\end{aligned}$$

Example (3) :

$x - 2$ subtracted from $x^2 - 5x$

$$\begin{aligned}
&= x^2 - 5x - (x - 2) \\
&= x^2 - 5x - x + 2 \\
&= x^2 - 6x + 2.
\end{aligned}$$

3. Subtraction—*continued.*

From these examples we see that the effect of using the brackets after the $-$ sign is to change all the signs in the expression being subtracted, after which the like terms are collected by the ordinary rules of addition. Remembering this, it is possible to do the work more conveniently by arranging the expressions one under the other as in Arithmetic; then changing the signs in the expressions to be subtracted and adding. This is the method which is usually used.

Thus the **fundamental rule of subtraction is: CHANGE THE SIGNS IN THE BOTTOM LINE AND ADD.** The change should be done mentally only and not actually on the paper.

The three examples already worked with brackets are shown below worked by the column method.

Example (1):

From $5a + 2b$ subtract $2a - 3b$.

$$\begin{array}{rr} 5a & +\ 2b \\ 2a & -\ 3b \\ \hline 3a & +\ 5b \\ \hline \end{array} \qquad \textbf{Answer.}$$

$5a - 2a = 3a$ and $2b + 3b = 5b$.

Example (2):

From $7x^2 - 5x + 6$ subtract $5x^2 - 3x + 2$.

$$\begin{array}{rrr} 7x^2 & -\ 5x & +\ 6 \\ 5x^2 & -\ 3x & +\ 2 \\ \hline 2x^2 & -\ 2x & +\ 4 \\ \hline \end{array} \qquad \textbf{Answer.}$$

$7x^2 - 5x^2 = 2x^2$;
$-5x + 3x = -2x$ and $6 - 2 = 4$.

Example (3):

From $x^2 - 5x$ subtract $x - 2$.

$$\begin{array}{rrr} x^2 & -\ 5x & \\ & x & -\ 2 \\ \hline x^2 & -\ 6x & +\ 2 \\ \hline \end{array} \qquad \textbf{Answer.}$$

As in addition, it is important to see that only like terms are put in the same vertical column. Also, care must be taken that the expressions are set down in their correct positions, with the one to be subtracted always below the one from which it is being subtracted.

A few more examples of subtraction are now given:—

Example (4):

Subtract $2a - b - 4d$ from $a - 3b + c$.

$$\begin{array}{rrrr} a & -\ 3b & +\ c & \\ 2a & -\ b & & -\ 4d \\ \hline -\ a & -\ 2b & +\ c & +\ 4d \\ \hline \end{array} \qquad \textbf{Answer.}$$

3. **Subtraction**—*continued.*

Example (5) :

Subtract $a^2 - 1 - 2a^4 - 3a + 5a^3$ from $3a^3 - 4a^4 + 6a^2 - 2$.

$$\begin{array}{rl} -4a^4 + 3a^3 + 6a^2 \qquad - 2 & \text{Re-arrange in descending order} \\ -2a^4 + 5a^3 + \ a^2 - 3a - 1 & \quad \text{and leave space for missing} \\ \hline & \quad \text{term } a. \\ -2a^4 - 2a^3 + 5a^2 + 3a - 1 & \textbf{Answer.} \\ \hline \end{array}$$

Example (6) :

From $5x - 8a - 2$ subtract $3x - 4a + 11$.

$$\begin{array}{rl} 5x - 8a - \ 2 & \\ 3x - 4a + 11 & \text{2nd expression underneath in this case.} \\ \hline 2x - 4a - 13 & \qquad \textbf{Answer.} \\ \hline \end{array}$$

EXERCISES.

Subtract :—

(1) $(3a + b - c - d)$ from $(6a - b + c - 3d)$.

(2) $(5x + 2y - 3z + 9)$ from $(6x - 3y - 4z + 7)$.

(3) $(-3ab + 2)$ from $5a^2 - 7ab - 12$.

(4) $(8 - 2x - 8x^2 - 2x^3)$ from $(3x - 4x^3 + 7x^2 - 9)$.

(5) the sum of $3a - b + 2c - 5d$ and $a + b - 2c + 3d$ from $6a - c - d$.

(6) $3x - y - z$ from the sum of $x + y - z$ and $3y - z$.

1. MULTIPLICATION.

(1) RULE OF SIGNS.

From Arithmetic, we know that $3 \times 2 = 6$, all being positive numbers. Let us now consider cases involving negative numbers, such as :—

-3×2. Obviously this means -3 taken twice, which is -6, i.e., $-3 \times 2 = -6$.

3×-2. Similarly, this means -2 taken three times, which is -6, i.e., $3 \times -2 = -6$.

-3×-2. Remembering that -2 is the opposite of $+2$, the answer to this must be the opposite of $-3 \times +2$. But we have seen that $-3 \times +2 = -6$, and, therefore, we can say that $-3 \times -2 = +6$. Thus we see that, when both the numbers multiplied together are positive, and also when both are negative, the answer is a positive number; but when one positive and one negative number are multiplied together the answer is a negative number. This may be expressed shortly in the following form as the rule of signs for multiplication :—

IN MULTIPLYING, LIKE SIGNS GIVE PLUS (+); UNLIKE SIGNS GIVE MINUS (–).

(2) RULE OF INDICES.

In a^2, a is raised to the second **power**; in a^3, to the third **power** and so on. The numbers 2 and 3 are each called an **index.** The plural of index is **indices.**

As was said in Lecture 1 :—

$a^2 = a \times a$ and $a^3 = a \times a \times a$.

$a^2 \times a^3 = a \times a \times a \times a \times a = a^5$.

Similarly $b^5 = b \times b \times b \times b \times b$ and $b^3 = b \times b \times b$.

$b^5 \times b^3 = b \times b \times b \times b \times b \times b \times b \times b = b^8$.

In both these cases we see that the index in the answer is equal to the sum of the indices in the two factors. This is usually expressed as follows, and forms the rule of indices for multiplication :—

IN MULTIPLYING POWERS OF THE SAME QUANTITY, ADD THE INDICES.

These two rules are the only ones to be observed in multiplying single terms, and various examples are given below :—

$-2a \times a = -2a^2$.

$7x^2 \times -2x = -14x^3$.

$a^2b \times ab^2 = +a^3b^3$ (add indices of a and b separately).

$-3x^2y \times -2xy^3 = +6x^3y^4$ (add the indices of x and y separately).

$a^2b^3c^4 \times ab^2c^3 = a^3b^5c^7$ (add the indices of a, b and c separately).

$-2x^2 \times 5y = -10x^2y$ (never add indices of different quantities x and y).

EXAMPLES.

When an expression containing two or more terms is multiplied by a single term, each term of the former is multiplied separately by the latter, according to the above rules, thus:—

Example (1):

Multiply $2a \quad 3b + 2c$ by -4.

$$-4\,(2a - 3b + 2c) = -8a + 12b - 8c.$$

Example (2):

Multiply $3a^2 - 2a + 5$ by $2a$.

$$2a\,(3a^2 - 2a + 5) = 6a^3 - 4a^2 + 10a.$$

Example (3):

Multiply $6a^3 - 4a^2 - 2a - 5$ by $7a^2$.

$$7a^2\,(6a^3 - 4a^2 - 2a - 5) = 42a^5 - 28a^4 - 14a^3 - 35a^2.$$

Example (4):

Multiply $2ab - 3bc - 4ac$ by $-3abc$.

$$-3abc\,(2ab - 3bc - 4ac) = -6a^2b^2c + 9ab^2c^2 + 12a^2bc^2.$$

When both expressions to be multiplied together contain two or more terms, the method used is very similar to that of long multiplication in Arithmetic. Thus, if we have to multiply $x + 4$ by $x + 3$, we first multiply $x + 4$ by x, then multiply $x + 4$ by $+3$, and finally add the two partial answers. The work is arranged thus:—

Example (5):

$$\begin{array}{llll} x & + 4 & & \\ x & + 3 & & \\ \hline x^2 & + 4x & & \quad x \text{ times } x + 4. \\ & 3x & + 12 & \quad 3 \text{ times } x + 4\text{; keep like terms} \\ \hline x^2 & + 7x & + 12 & \quad \text{in same column for adding.} \\ \hline \end{array}$$

Other examples involving $-$ signs are now given; the method is the same.

Example (6):

$$\begin{array}{llll} x & + 5 & & \\ x & - 2 & & \\ \hline x^2 & + 5x & & \quad x \text{ times } x + 5. \\ & - 2x & - 10 & \quad -2 \text{ times } x + 5. \\ \hline x^2 & + 3x & - 10 & \\ \hline \end{array}$$

Examples—*continued.*

Example (7):

$$\begin{array}{l} x - 5 \\ x + 3 \\ \hline x^2 - 5x \\ \quad\;\; 3x - 15 \\ \hline x^2 - 2x - 15 \\ \hline \end{array}$$

Example (8):

$$\begin{array}{l} x - 7 \\ x - 4 \\ \hline x^2 - 7x \\ \quad - 4x + 28 \\ \hline x^2 - 11x + 28 \\ \hline \end{array}$$

The same method is also applied to the multiplication of still longer expressions, as shown below :—

Example (9):

$$\begin{array}{rrrrrl} x^3 & - 3x^2 & + 3x & - 1 & & \\ & & x & - 2 & & \\ \hline x^4 & - 3x^3 & + 3x^2 & - x & & x \text{ times top line.} \\ & - 2x^3 & + 6x^2 & - 6x & + 2 & -2 \text{ times top line.} \\ \hline x^4 & - 5x^3 & + 9x^2 & - 7x & + 2 & \\ \hline \end{array}$$

Note.—It is always helpful to arrange both multiplicand and multiplier in descending powers of the same symbol.

To show that two expressions such as $x + 3$ and $x - 2$ are to be multiplied together, they are usually written $(x + 3)\ (x - 2)$. Thus, when two brackets come together with no signs between them, it indicates that the whole of one must be multiplied by the whole of the other.

EXERCISES.

MULTIPLY.

1. $4x^2$ by $2x^3$.
2. $- a^{11}$ by $- a^3$.
3. $- 2a^2b^3c$ by $3a^3bc^2$.
4. $x^3 - 3x^2y + 3xy^2 - y^3$ by $- 3x^2$.

Exercises—*continued.*

Multiply—*continued.*

5. $3ab + 2ac - bc$ by $-12a^6b^4c^3$.
6. $5a^3 - ab^4c^3 + 9b^5c^2$ by $-12a^6b^4c^3$.
7. $x - 11$ by $x - 7$.
8. $x + 2y$ by $x - 3y$.
9. $x + 6$ by $x - 3$.
10. $2x - 3$ by $3x - 4$.
11. $2 - 3x$ by $3 - 2x$.
12. $x^3 + 2x^2 + x - 4$ by $x - 2$.

2. **DIVISION.**

(1) **RULE OF SIGNS.**

In the section on multiplication we considered four different combinations of signs, namely :—

(1) $+3 \times +2 = +6$.
(2) $-3 \times +2 = -6$.
(3) $+3 \times -2 = -6$.
(4) $-3 \times -2 = +6$.

If we now reverse the process and divide the product by the first factor in each case, the answers will be the second factor, thus :—

(1) $+6 \div +3 = +2$.
(2) $-6 \div -3 = +2$.
(3) $-6 \div +3 = -2$.
(4) $+6 \div -3 = -2$.

Thus we see that when divisor and dividend have the same sign (Nos. 1 and 2) the answer is positive, and when they have opposite signs (Nos. 3 and 4) the answer is negative. **The rule of signs for division** is therefore the same as for multiplication, namely :—

IN DIVIDING, LIKE SIGNS GIVE PLUS (+) ; UNLIKE SIGNS GIVE MINUS (−).

(2) **RULE OF INDICES.**

In multiplication we also learned that the indices of the factors must be added, thus :—

$$a^2 \times a^3 = a^5$$
$$b^3 \times b^5 = b^8, \text{ and so on.}$$

If we reverse the process and divide the product by one of the factors, the answer will be the other factor, thus :—

$$a^5 \div a^2 = a^3 \text{ and } a^5 \div a^3 = a^2.$$
$$b^8 \div b^3 = b^5 \text{ and } b^8 \div b^5 = b^3.$$

2. **Division**—*continued.*

In each case we see that the index of the answer is equal to the difference of the indices of the dividend and the divisor. This may be expressed as follows as **the rule of indices for division** :—

IN DIVIDING POWERS OF THE SAME QUANTITY, SUBTRACT THE INDICES.

We now give several examples embodying both these rules :—

$$\begin{aligned} 3x &\div -3 = -x. \\ 7abc &\div -7a = -bc. \\ a^2 &\div -a = a. \\ -x^3 &\div -x = +x^2. \\ 21a^4 &\div -3a^2 = -7a^2. \\ 12a^3x^4 &\div 4ax = 3a^2x^3. \\ -54a^2bc &\div -6abc = +9a. \end{aligned}$$

EXAMPLES.

When an expression containing two or more terms is to be divided by a single term, each term of the former is divided separately by the latter according to the above rules, thus :—

Example 1 :

Divide $3a - 9b$ by -3.

$$\frac{3a - 9b}{-3} = -a + 3b. \quad \textbf{Answer.}$$

Example 2 :

Divide $9a^3b - 21ab^3$ by $-3ab$.

$$\frac{9a^3b - 21ab^3}{-3ab} = -3a^2 + 7b^2. \quad \textbf{Answer.}$$

Example 3 :

Divide $7a^4b^2 + 35a^3b^4 - 21a^3b^3$ by $7a^3b^2$

$$\frac{7a^4b^2 + 35a^3b^4 - 21a^3b^3}{7a^3b^2} = a + 5b^2 - 3b. \quad \textbf{Answer.}$$

When the divisor contains more than one term, the method used is very similar to that of long division in Arithmetic. The divisor is placed to the left of the dividend, on the same line, and the answer may be written either above the dividend or to the right of it.

Examples *continued.*

Example 4:

Suppose we are dividing $x^2 + 5x + 6$ by $x + 3$.

Setting out :—

$$\begin{array}{r|l} & x + 2 \quad \textbf{Answer.} \\ \hline x + 3) & x^2 + 5x + 6 \\ & x^2 + 3x \\ \hline & 2x + 6 \\ & 2x + 6 \\ \hline \end{array}$$

Method.

Begin by dividing 1st term of divisor into 1st term of dividend, i.e., x into x^2 goes x times. Place this in the answer; then multiply the whole divisor by x and place the product under the like terms in the dividend. Subtract, and bring down next term. Now divide 1st term of this remainder by 1st term of divisor, i.e., x into $2x$ goes $+ 2$ times. Place $+ 2$ in the answer, then multiply divisor by $+ 2$. Place this result $(2x + 6)$ under the last remainder, and subtract. In this case no remainder is left, so that the answer is $x + 2$ exactly.

Now study the other examples of the same method.

It is most important in division that both divisor and dividend be arranged in descending order of same letter. Also, in the dividend, spaces must be left for any missing terms.

Example 5:

Divide $x^2 - 8x + 15$ by $x - 5$.

$$\begin{array}{r|l} & x - 3 \quad \textbf{Answer.} \\ \hline x - 5) & x^2 - 8x + 15 \\ & x^2 - 5x \\ \hline & -3x + 15 \\ & -3x + 15 \\ \hline \end{array}$$

x into x^2 goes x times. x times $x - 5$.

Subtracting, $- 8x + 5x = - 3x$. Bring down next term. x into $- 3x$ goes -3 times. -3 times $x - 5$.

Example 6:

Divide $10x^2 - 14x - 12$ by $2x - 4$.

$$\begin{array}{r|l} & 5x + 3 \quad \textbf{Answer.} \\ \hline 2x - 4) & 10x^2 - 14x - 12 \\ & 10x^2 - 20x \\ \hline & 6x - 12 \\ & 6x - 12 \\ \hline \end{array}$$

$2x$ into $10x^2$ goes $5x$ times. $5x$ times $2x - 4$.

Subtracting, $- 14x + 20x = + 6x$. $2x$ into $6x$ goes 3 times. 3 times $2x - 4$.

Examples—*continued.*

Example 7 :

Divide $x^2 - 16$ by $x + 4$.

$$
\begin{array}{rl}
 & \underline{x \quad - 4 \qquad\quad} \\
x + 4) & x^2 \qquad\quad - 16 \\
 & \underline{x^2 + 4x \qquad\quad} \\
 & \qquad 4x - 16 \\
 & \qquad 4x \quad 16 \\
 & \qquad \underline{\quad}
\end{array}
$$

Answer.

In this case a space must be left in dividend for missing term x.

Example 8 :

Divide $8x^3 - 6x^2 + 3x - 18$ by $2x - 3$.

$$
\begin{array}{rl}
 & \underline{4x^2 + 3x + 6 \qquad\qquad} \\
2x - 3) & 8x^3 - \; 6x^2 + 3x - 18 \\
 & \underline{8x^3 - 12x^2 \qquad\qquad} \\
 & \qquad\;\; 6x^2 + 3x \\
 & \qquad\;\; \underline{6x^2 - 9x \qquad} \\
 & \qquad\qquad\quad 12x - 18 \\
 & \qquad\qquad\quad \underline{12x - 18}
\end{array}
$$

Answer.

$2x$ into $8x^3$ goes $4x^2$ times.

$2x$ into $6x^2$ goes $+ 3x$ times.

$2x$ into $12x$ goes $+ 6$ times.

Example 9 :

Divide $ax^2 - 7ax - 5cx + 35c$ by $x - 7$.

$$
\begin{array}{rl}
 & \underline{ax \quad - 5c \qquad\qquad\qquad} \\
x - 7) & ax^2 - 7ax - 5cx + 35c \\
 & \underline{ax^2 - 7ax \qquad\qquad\qquad} \\
 & \qquad\qquad - 5cx + 35c \\
 & \qquad\qquad \underline{- 5cx + 35c}
\end{array}
$$

Answer.

$ax^2 \div x = ax$.

Bring down next 2 terms in such a case as this.
$- 5cx \div x = - 5c$.

EXERCISES.

DIVIDE.

1. $- 4a^3$ by $- 2a$.
2. $24p^2qr^2$ by $- 6p^2qr$.
3. $- 28a^6b^4$ by $4a^3b^3$.
4. $96a^7b^6$ by $4a^2b^2$.
5. $y^2 - 6y$ by $- y$.
6. $7x^4 + 9x^3$ by $- x^3$.

Exercises—*continued.*

Divide—*continued.*

7. $6x^7y^9z^3 - 5x^5y^7z^6$ by $x^3y^4z^2$.
8. $4a^4b^8 - 8a^5b^6 - 28a^6b^4$ by $-4a^3b^3$.
9. $a^2 - 15a + 54$ by $a - 9$.
10. $2x^2 + 3x - 2$ by $x + 2$.
11. $9 - 6x + x^2$ by $3 - x$.
12. $3a^2 - 8a + 4$ by $3a - 2$.
13. $3x + 8x^3 - 18 - 6x^2$ by $2x - 3$.

REVISION EXERCISES.

1. Divide the sum of $x(x - 3)$ and $2(3 - x)$ by $x - 2$.
2. Divide the product of $(3x - 6a)$ and $(5x - 15a)$ by $(x - 2a)$.
3. Simplify $\{6x(x - 1) + 5(x - 3)\} \div (3x - 5)$.
4. Simplify $(3x + 9)(7x - 21) \div (x - 3)$.
5. Divide the sum of $x^3 + 1$ and $3x(x + 1)$ by $(x + 1)$.
6. Find the product of $2x^2 - 9x - 5$ and $x - 1$, and divide your result by $2x + 1$.
7. Divide the sum of $2x - 7 - 3x^2$, $5x^2 + 1 - 3x$ and $7 - 4x + 2x^2$ by $(4x - 1)$.
8. Divide $5(x - 1)(x + 1) + 3x(3x + 1)$ by $7x - 5$.
9. Multiply $5x^2 - 2(x^2 - a)$ by $2a - 3(a - 2x^2)$.
10. If x stands for $ax^2 + 5bx + 5c$ and y for $ax^2 - 6bx - 6c$, find the value of $6x + 5y$.
11. Find the value of $(x - 2y) \times (x + 2y) \times (x - 2y)$.

TRAINING COURSE.

GROUND DUTIES CADETS.

STAGE 1.

Algebra. **1B/2b/1.**

REVISION OF PRELIMINARY COURSE.

1. Revise Lecture 1, 1B/1 Algebra.

2. Revision will include **Symbolic Expression, Substitution, Positive and Negative Quantities.**

1. SUBSTITUTION.

If we know the numerical values of the symbols in an expression, we can find the numerical value of the whole expression by substituting the given numbers in the place of the symbols.

EXAMPLES.

(i) If $x = 5$, $7x = 7 \times 5 = 35$.

(ii) If $x = 2$, $y = 3$, $xy = 2 \times 3 = 6$.

(iii) If $a = 3$, $a^2 = 3 \times 3 = 9$.
$a^3 = 3 \times 3 \times 3 = 27$.

(iv) If $x = 0$, $y = 1$, $z = 2$

$$\begin{aligned} 5x + 8y - 3z &= (5 \times 0) + (8 \times 1) - (3 \times 2) \\ &= 0 + 8 - 6 \\ &= 2. \end{aligned}$$

(v) If $g = 32$, $t = 2$,

$$\tfrac{1}{2}\, gt^2 = \tfrac{1}{2} \times 32 \times 2^2 = \tfrac{1}{2} \times 32 \times 4 = 64.$$

N.B.—The index 2 refers only to the symbol t.

(vi) If $a^2 = b^2 + c^2$, find a^2 when $b = 3$, $c = 4$

$$a^2 = 3^2 + 4^2 = 9 + 16 = 25$$

i.e., $a^2 = 25$.

(vii) If $a = 2$, $b = 0$, $a^b = 2^0 = 1$.

2. POSITIVE AND NEGATIVE QUANTITIES.

The subtraction of 10 from 5 cannot be performed arithmetically, i.e., the expression $5 - 10$ has no arithmetical meaning. Such an expression can, however, be explained algebraically and a term such as -5 can have a meaning attached to it.

EXAMPLES.

(i) If a business man, in the course of trading, makes a gain of £50 and then loses £25, the result of these transactions represents a gain of £25.

If he had gained £50 and then lost £50, he would have been in the same position as at the beginning of the transactions.

If however, he were to gain £25 and then lose £50, the result of the trading would be a loss of £25.

The first two cases can be represented :—

$$£50 - £25 = +\ £25.$$

$$£50 - £50 = 0.$$

Examples—*continued.*

The loss of £25 in the third case can be represented algebraically as $-$ £25, or, in the form of an operation, as follows :—

$$£25 - £50 = -£25$$

$+$ 25 is called **a positive number** and $-$ 25 **a negative number.**

(ii) If a town O lies on a road running due East and West, as in the diagram, distances to the East of O can be distinguished from those to the West by using the positive and negative notation.

WEST $\underset{B}{\cdot}\overset{\text{2 miles}}{\rule{2em}{0.4pt}}\underset{O}{\cdot}\overset{\text{2 miles}}{\rule{2em}{0.4pt}}\underset{A}{\cdot}$ **EAST**

If the distances to the East are denoted by positive numbers, then those to the West may be represented by negative numbers.

Thus the distance OA may be expressed, $+$ 2 miles and the distance OB, $-$ 2 miles.

Many other concrete examples can be found, showing that some quantities are capable of existing in two opposite states. In Algebra, it is convenient to employ the $+$ and $-$ signs, not only to indicate addition and subtraction, but also to indicate the state or quality of the quantities to which they are attached, i.e., if a symbol $+a$ denoted a certain quality, e.g., degrees **above** zero, then, by altering the sign to give $-a$, we should imply the opposite quality, namely degrees **below** zero.

(iii) If a man loses £8 one day and then loses £5 the next day also, obviously he has lost £13 altogether. We can say, therefore, in the form of a relation, that

$$-£8 - £5 = -£13.$$

In the discussion above, we have largely taken concrete quantities, £'s, distances, &c. The same reasoning can be applied to Algebraic symbols.

Thus we can have,

$$\begin{aligned} 50a - 25a &= +25a \text{ (or just } 25a) \\ 50a - 50a &= 0 \\ 25a - 50a &= -25a \\ -8a - 5a &= -13a. \end{aligned}$$

REVISION.

Definitions and Rules :

1. **A power** is the product obtained when any quantity is multiplied by itself one or more times. e.g., Third power of $a = a \times a \times a$. Sixth power of $b = b \times b \times b \times b \times b \times b$.

2. **An index** is the small number, written above the symbol, which indicates how many equal factors are multiplied together in the power ; e.g., $a \times a \times a = a^3$.

$$b \times b \times b \times b \times b \times b = b^6.$$

3. Positive and negative numbers.

In Algebra, the signs $+$ and $-$ are employed, not only to indicate the processes of addition and subtraction, but also to denote the quality or state of a quantity capable of existing in two opposite states.

e.g., if $+ x$ miles represents a distance due North of a certain locality, then $- x$ miles represents a similar measurement due South.

If a given quantity is subtracted from a smaller like quantity, the result is expressed as a negative quantity,

$$\text{e.g., } 3x - 5x = - 2x.$$

4. Rules for Addition.

(*a*) Only like terms can be added together into a single term.

(*b*) If the signs of the terms are the same, add the co-efficients and take the same sign.

(*c*) If the signs of the terms are different, subtract the co-efficients and take the sign of the greater.

5. Rule for Subtraction.

Change the signs in the bottom line, and add.

6. Rules for Multiplication.

(*a*) Like signs, multiplied together, give $+$, unlike signs give $-$.

(*b*) Add the indices of powers of the same quantity.

7. Rules for Division.

(*a*) In division, like signs give $+$, unlike signs give $-$.

(*b*) In dividing powers of the same quantity, subtract the indices.

REVISION EXERCISES.

1. Express b pounds in quarters; in cwt.; in tons.
2. Express c seconds in minutes; in hours.
3. If I walk x miles per hour, how far do I walk in a hours?
4. If $a = 5$, $b = 3$, $c = 2$, $x = 0$, $y = 4$, find the value of:—

   (*a*) $4a^2$ (*c*) $3a^2b^3c^2x$ (*e*) $\frac{3}{5}a^2bc$

   (*b*) $6acx$ (*d*) $5xy + 2abc$
5. Find the value of $a^2 - 3b^2 - 2ac$ when $a = 0$, $b = 2$, $c = 1$.
6. Find the value of $x^2 + 2x + 1$ when $x = 2$, and also when $x = - 2$.
7. Simplify: $5\ (x - 3) - 3\ (x - 2) - (2x - 9)$.
8. Simplify: $\dfrac{5x - 10}{5} - \dfrac{7x + 21}{7} + \dfrac{3x - 9}{3}$.
9. Simplify: $x^3 - (x - 2) + 3\ (x^2 - 2 - 5x)$.

Revision Exercises—*continued.*

10. Multiply:

(i) $-\frac{3}{4}a$ by $\frac{4}{3}x$.

(ii) $\frac{2}{3}ab^2c$ by $-\frac{9}{2}a^2bc^2$.

(iii) $x + 2y$ by $x - 3y$.

(iv) $2x - 5y$ by $3x - 2y$.

(v) $5x + y$ by $5x - y$.

11. Divide:

(i) $12ab^2c^3 - 16a^2b^3c$ by $-4abc$.

(ii) $27x^4y^5z^6 - 45x^5y^4z^5 + 54x^6y^7z^4$ by $9x^3y^3z^2$.

(iii) $10x^2 - 14x - 12$ by $2x - 4$.

(iv) $3x^2 - xy - 14y^2$ by $x + 2y$.

(v) $25x^2 - 16$ by $5x - 4$.

12. If $u = 10$, $g = 32$, $t = 3$, $s = 174$, find the values of:

(i) $ut + \frac{1}{2}gts$.

(ii) $u^2 + 2gs$.

(iii) $u + gt$.

13. If $a = -1$, $b = 2$, $c = 0$, find the values of:

(i) $b^2 - 3a^2 + 2c$.

(ii) $a^3 - b^3 - 3b^c$.

THE USE OF FORMULAE.

1. **A formula** is the relationship between two or more quantities expressed entirely by means of symbols and numbers.

Thus :—

$A = l \times b$ expresses the relation between the area, the length, and the breadth of any rectangle ;

$A = \pi r^2$ expresses the relation between the area and the radius of a circle ; π has the value of $\frac{22}{7}$ approximately ;

$V = \pi r^2 h$ expresses the relation between the volume, the radius and the height of a cylinder.

In Lecture 3 of 2/1 Science, certain fundamental equations of motion were established from first principles, viz.,

Body having a uniform velocity.

$$s = vt$$

Body starting from rest. *Body starting with initial velocity.*

$$a = \frac{v}{t} \qquad\qquad a = \frac{v-u}{t}$$

$$s = \frac{v}{2}\, t \qquad\qquad s = \frac{u+v}{2}\, t$$

Other important formulae may be developed as follows :—

(*a*) *Body starting from rest* (*b*) *Body starting with initial velocity.*

(a)

$$a = \frac{v}{t}$$

or $v = at$

but $s = \frac{v}{2}\, t$

$$= \frac{at}{2}\, t$$

$$s = \tfrac{1}{2}\, at^2$$

Again $v^2 = a^2t^2$

$$= 2a\ (\tfrac{1}{2}at^2)$$

$$= 2as$$

(b)

$$a = \frac{v-u}{t}$$

or $v = u + at$

but $s = \frac{u+v}{2}\, t$

$$= \frac{u + u + at}{2}\, t$$

$$s = ut + \tfrac{1}{2}at^2$$

Again $v^2 = u^2 + 2uat + a^2t^2$

$$= u^2 + 2a\ (ut + \tfrac{1}{2}at^2)$$

$$u^2 + 2as$$

Hence, we have the following list of formulae which can be used in solving practical problems. These formulae, or preferably their method of derivation, should be memorized.

The Use of Formulae—*continued.*

Body starting from rest.	**Body starting with initial velocity.**
$v = at.$	$v = u + at.$
$s = \frac{v}{2} t.$	$s = \frac{u + v}{2} t.$
$s = \frac{1}{2}at^2.$	$s = ut + \frac{1}{2}at^2.$
$v^2 = 2as.$	$v^2 = u^2 + 2as.$

The symbol a is replaced by g for bodies acted upon by gravity.

In solving problems, care should be taken to use consistent units for the quantities u, v, a, t and s.

2. It will be seen that a formula is usually expressed with one symbol on the left and the remainder on the right. This single symbol on the left is generally called **the subject of the formula.**

A given formula may be rearranged so as to make any other symbol the subject, for a formula can be expressed in as many different ways as there are symbols in it. This can be done by using the rules of equations already learnt, i.e., to change the position of any symbol, multiply or divide both sides by that symbol, or transpose it from one side to the other, &c.

Thus :—

(i) $A = l \times b.$

To make b the subject, divide both sides of the equation by l,

$$\frac{A}{l} = b, \text{ or } b = \frac{A}{l}.$$

To make l the subject, divide both sides of the equation by b,

$$\frac{A}{b} = l, \text{ or } l = \frac{A}{b}.$$

(ii) $A = \pi r^2.$

To make r the subject, divide both sides of the equation by π,

$$\frac{A}{\pi} = r^2, \text{ or } r^2 = \frac{A}{\pi} \quad \text{i.e., } r = \sqrt{\frac{A}{\pi}}.$$

The Use of Formulae—*continued.*

To make π the subject, divide both sides of the equation by r^2,

$$\frac{A}{r^2} = \pi, \text{ or } \pi = \frac{A}{r^2}.$$

(iii) $S = \frac{n}{2}(a + l)$.

Using a similar procedure, it can be shown that $n = \dfrac{2S}{a + l}$.

To make l the subject, divide both sides of the equation by n, and multiply both sides by 2,

$$\frac{2S}{n} = a + l$$

$$\therefore a = \frac{2S}{n} - l, \text{ or } l = \frac{2S}{n} - a.$$

(iv) If $v^2 = u^2 + 2as$

then, $2as = v^2 - u^2$

$$s = \frac{v^2 - u^2}{2a}, \text{ or } a = \frac{v^2 - u^2}{2s}$$

and, $u^2 = v^2 - 2as$ i.e., $u = \sqrt{v^2 - 2as}$.

(v) Again, if $s = ut + \frac{1}{2}at^2$

then, $\frac{1}{2}at^2 = s - ut$

$$at^2 = 2(s - ut)$$

$$a = 2\frac{(s - ut)}{t^2}.$$

3. When the numerical values of all the symbols except one in a formula are known, **the value of that symbol** may be obtained by substitution.

Thus :—

Example 1.

The circumference C of a circle is given by the formula

$$C = 2\pi r,$$

where r is the radius and $\pi = 3\frac{1}{7}$ or $\frac{22}{7}$. Find the radius of a circle whose circumference is 22 in.

N.B.—This may be performed either by—

(1) substituting direct into the formula as it is, or

(2) by changing the subject to r and then substituting.

(1) $C = 2\pi r$

$$22 = 2 \times \frac{22}{7} \times r$$

$$22 = \frac{44}{7} \times r$$

i.e., $r = \dfrac{1 \times 7}{2}$

$r = \frac{7}{2}$ or $3\frac{1}{2}$ inches.

(2) $C = 2\pi r$

Dividing both sides by 2π

$$\frac{C}{2\pi} = r$$

i.e., $r = \dfrac{C}{2\pi} = \dfrac{22}{2 \times \frac{22}{7}}$

$= \frac{1}{2} \times \frac{7}{1}$

$= \frac{7}{2}$ or $3\frac{1}{2}$ inches.

The Use of Formulae—*continued.*

Example 2.

If a body starting from rest falls freely under gravity for t seconds, the distance (s) in feet, through which it falls, is given by the formula $s = \frac{1}{2}gt^2$, where $g = 32$ ft. per sec^2. How far will such a body fall in 6 seconds?

$$\begin{aligned} s &= \tfrac{1}{2}gt^2 \\ &= \tfrac{1}{2} \times 32 \times 36 \text{ ft.} \\ &= 576 \text{ ft.} \end{aligned}$$

Example 3.

Find how long a body, starting from rest and falling freely under gravity, takes to fall 144 ft. ($g = 32$ ft. per sec^2.)

$$\begin{aligned} s &= \tfrac{1}{2}gt^2 \\ \therefore 144 &= \tfrac{1}{2} \times 32t^2 \\ 16t^2 &= 144 \\ t^2 &= 9 \\ t &= \sqrt{9} \\ &= 3 \text{ secs.} \end{aligned}$$

$$\begin{aligned} \text{or, } s &= \tfrac{1}{2}gt^2 \\ 2s &= gt^2 \\ t^2 &= \frac{2s}{g} \\ &= \frac{2 \times 144}{32} = 9 \;\therefore\; t = \sqrt{9} \\ &= 3 \text{ secs.} \end{aligned}$$

N.B.—The following examples show how an equation of motion can be selected to fit a given problem.

Example 4.

A body, moving in a straight line with a uniform acceleration of 3 ft./sec.^{2}, has, at a certain instant, a velocity of 5 ft. per sec. What is its velocity after 10 seconds?

Set down the values that are given and wanted—

$a = 3$ ft./sec.^{2}, $u = 5$ ft./sec., $t = 10$ seconds, $v =$?

The formula containing the symbols concerned is—

$$v = u + at$$

$$v = 5 + 3 \times 10 = 5 + 30 = \mathbf{35 \text{ ft./sec.}}$$

How far did the body travel in the time interval concerned?

We can choose one of several formulae for this part of the problem.

e.g., $s = \frac{u+v}{2}t$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$.

Select the simplest—

$$\begin{aligned} s &= \frac{u+v}{2}t \\ &= \frac{5+35}{2}10 = \frac{40}{2} \times 10 = \mathbf{200 \text{ ft.}} \end{aligned}$$

The Use of Formulae—*continued.*

Example 5.

To rise to 1000 ft., what must be the initial velocity of a missile projected vertically upwards? ($g = 32 \cdot 2$ ft./sec.^{2})

$S = 1000$ ft., $v = 0$ (at the highest point, the velocity of a body projected vertically upwards is zero)

$g = -32 \cdot 2$ ft./sec.^{2} (for a **rising** body g is negative)

$u = ?$

The best formula to take is, $v^2 = u^2 + 2gs$

$$\begin{aligned} \text{i.e., } 0 &= u^2 + 2 \times -32 \cdot 2 \times 1000 \\ \text{i.e., } 0 &= u^2 - 64{,}400 \\ \text{or } u^2 &= 64{,}400 \\ u &= \sqrt{64{,}400} \\ &= 254 \text{ ft./sec. (approx.)} \end{aligned}$$

Example 6.

It can be shown, from Newton's Law of Universal Gravitation, that the mass E of the earth is given by the formula:

$$E = \frac{g\ R^2}{K}$$

where g is the value of the acceleration due to gravity, R is the radius of the earth and K is a constant equal to $6 \cdot 66 \times 10^{-8}$ (*C.G.S.*).

If $g = 981$ cm/sec.^{2}, and $R = 6 \cdot 37 \times 10^8$ cm, find E.

$$\begin{aligned} E &= \frac{981 \times 6 \cdot 37 \times 10^8 \times 6 \cdot 37 \times 10^8}{6 \cdot 66 \times 10^{-8}} \\ &= \frac{981 \times (6 \cdot 37)^2 \times (10^8)^3}{6 \cdot 66} \\ &= 5 \cdot 98 \times 10^{27} \text{ grams.} \end{aligned}$$

In tons, the mass of the earth is 6×10^{21} tons approximately.

(**N.B.**—Details of calculation omitted.)

Example 7.

The volume (V) of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$, where r is the radius, and $\pi = 3\frac{1}{7}$. Find the volume of a sphere whose radius is $10\frac{1}{2}$ in.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} \text{ cub. in.} \\ &= 4851 \text{ cub. in.} \end{aligned}$$

EXERCISES.

1. (*a*) The area A of a triangle whose base is b and whose altitude is h is given by $A = \frac{1}{2}bh$. Rearrange this formula to make b the subject; and again to make h the subject.

(*b*) Find the area of a triangle whose base is 8 in. long and whose altitude is 5 in.

(*c*) A triangle whose area is 18 sq. ft. stands on a base of 8 ft. What is its altitude ?

2. (*a*) A bomb drops from an aircraft and strikes the ground 12 secs. later. At what height is the aircraft flying ? (Take $g = 32$ ft./sec.^{2}). Formula is $s = \frac{1}{2}gt^2$.

(*b*) How long will it take a bomb to fall from an aircraft flying 1600 ft. above the target ?

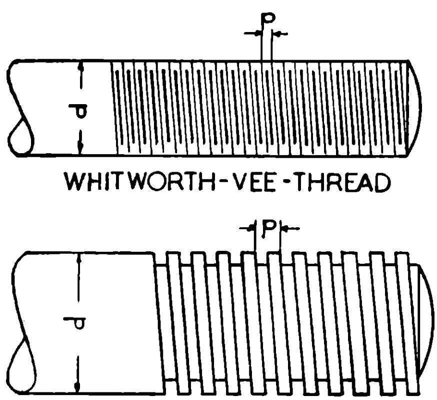
3. (*a*) The volume V of a cylinder is given by $V = \pi r^2 h$ where $\pi = 3\frac{1}{7}$, r is the radius of the base and h is the height. Rearrange this formula to make h the subject; and again to make r the subject.

(*b*) Find the volume of a cylindrical water tank whose base is 3 ft. in radius and whose height is $5\frac{1}{4}$ ft.

(*c*) A cylinder having a volume of 693 cubic ft. stands on a base of radius 7 ft. What is its height ?

4. The pitch p of a Whitworth Vee screw thread (Fig. 1) is given by $p = 0{\cdot}08d + 0{\cdot}04$, and the pitch of a square thread by $p = 0{\cdot}15d + 0{\cdot}08$.

Find the pitch when d is (i) 0·875 in., (ii) 1·125 in., (iii) 1·5 in.



SQUARE THREAD

Fig. I.

5. If n denotes the number of threads per inch length of screw, then $n = \frac{1}{p}$, where p is the pitch. Find n for screws (i) $\frac{7}{8}$ in., (ii) 1 in., (iii) $1\frac{1}{2}$ in., (iv) 0·625 in. diameter.

Exercises—*continued.*

6. Find the volume of a sphere of a radius 2 ft. Formula is $V = \frac{4}{3} \pi r^3$.

7. The weight w in lb. of a square beam of oak is given by the formula $w = 45a^2l$, where l is its length in feet, and a feet is the edge of its square end. Find the weight of a beam of oak 20 ft. long and 1 ft. square.

8. The formula for finding the radius of action of an aircraft is :—

$R = \frac{POH}{O + H}$ where R = radius of action in miles, P = "petrol hours," O = ground speed on outward track, H = ground speed on homeward track. If $P = 6$, $O = 142$ m.p.h., $H = 132$ m.p.h., what is R ?

How many gallons of fuel must be placed in an aircraft, which consumes 6 pints of petrol per minute, to give it a radius of action of 450 miles, if the ground-speed out is 120 m.p.h. and the ground-speed home is 144 m.p.h. ?

Express this formula so that P is the subject.

9. The indicated horse-power of an engine is given by the formula :—

$$\text{H.P.} = \frac{2\ \text{PLAN}}{33000}$$

where P = pressure in lb. per sq. in., A = area of piston in sq. in., N = the number of strokes per min., L = length of stroke in feet.

If $r = 5$ inches, $P = 130$ lb. per sq. in., $L = 1{\cdot}5$ feet, and the number of strokes is 200 per minute, what is the H.P. ? ($A = \pi r^2$, $\pi = \frac{22}{7}$.)

10. A bullet projected vertically upwards rises to 2500 ft. With what velocity was it projected ?

11. A body, moving at a certain instant with a velocity of 4 ft./sec., has a velocity of 14 ft./sec. six seconds later. What was its acceleration, assuming it was uniform during this interval of time ?

12. The resistance, which a flat plate offers to an airflow (the plate being at right angles to the airflow), is given by the formula :—

$$R = K\ s\ AV^2.$$

Find R when $K = 0{\cdot}6$, $s = 0{\cdot}00248$, $A = \frac{1}{3}$, and $V = 88$.

13. A parachutist drops 1000 feet before the parachute operates. At what velocity will he be falling at that moment ?

CO-ORDINATES AND STRAIGHT LINE GRAPHS.

1. CO-ORDINATES.

Draw two straight lines $X'OX$ and $Y'OY$, cutting each other at right angles.

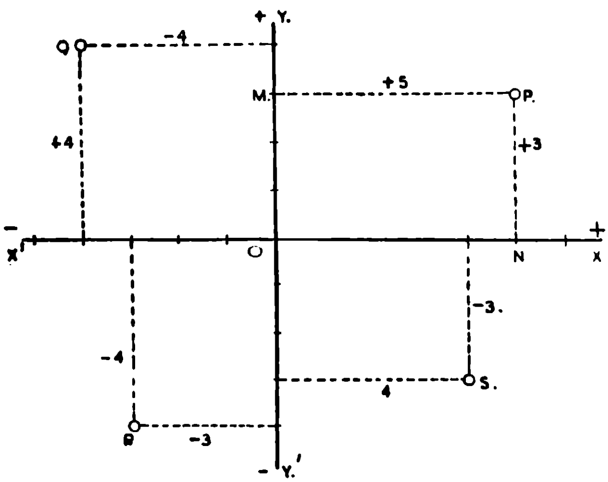


Fig. 1.

Let P be any other point on the page ; and draw PN and PM perpendicular (i.e., at right angles) to $X'OX$ and $Y'OY$ respectively.

$$\text{Let } PM = x$$
$$\text{and } PN = y.$$

The values of x and y fix the position of the point P, with reference to the straight lines $X'OX$, $Y'OY$. If the values of x and y are given, it is possible to plot the position of the point P. Note that $PM = ON = x$, $PN = OM = y$.

For example, if $x = 5$ units and $y = 3$ units, along OX, measure $ON = 5$ units and, along OY, measure $OM = 3$ units. The perpendiculars from M and N meet at P.

$$\text{Since } PM = ON = 5$$
$$\text{and } PN = OM = 3$$
$$P \text{ is the point required.}$$

The two lines $X'OX$ and $Y'OY$ are called the **axes.** $X'OX$ is termed the x-axis, $Y'OY$, the y-axis. The point O where they cut is termed the **origin,** and the distances x and y are called the **co-ordinates** of P. x is called the **abscissa,** and y the **ordinate,** of P.

The abscissa of a point, therefore, means its distance from the vertical axis or y-axis, and the ordinate means its distance from the horizontal or x-axis.

1. **Co-ordinates**—*continued.*

P is described as the point (x, y). In the specific case above P is the point (5, 3). In giving the co-ordinates of a point, the **abscissa x is always given first.**

It is necessary to distinguish between distances on opposite sides of each axis, and this is done by means of + and − signs.

Lines drawn in directions along or parallel to OX, OY are considered positive; lines drawn along or parallel to the directions OX', OY' are considered negative. In other words, measurements to the right of the y-axis are **positive** and those to the left of it **negative.** Measurements upwards from the x-axis are **positive,** and those downwards from it **negative.**

Thus, in Fig. 1, above :-

The co-ordinates of P are $x = 5$, $y = 3$, i.e., P is the point (5, 3).

The co-ordinates of Q are $x = -4$, $y = 4$, i.e., Q is the point $(-4, 4)$.

The co-ordinates of R are $x = -3$, $y = -4$, i.e., R is the point $(-3, -4)$.

The co-ordinates of S are $x = 4$, $y = -3$, i.e., S is the point $(4, -3)$.

The simplest **method** for **plotting** points is shown in the following examples (see Fig. 1):—

(*a*) To plot the point (5, 3).

Along OX measure $ON = 5$ units; then vertically along a line drawn from N in the positive direction, NP, parallel to OY, measure $NP = 3$ units.

Then P is the required point.

(*b*) To plot the point $(-4, 4)$.

From the origin, measure, along the x-axis, 4 units to the left, i.e., in the negative direction, and then 4 units upwards in the positive direction parallel to OY.

Then Q is the required point.

(*c*) To plot $(-3, -4)$.

From the origin, measure 3 units along the x-axis to the left (negative), then 4 units downwards (negative) parallel to OY'.

Then R is the required point.

(*d*) To plot $(4, -3)$.

From the origin measure 4 units to the right (positive) then 3 units downwards (negative) parallel to OY'.

Then S is the required point.

2. GRAPHS OF ALGEBRAIC EXPRESSIONS.—STRAIGHT LINE GRAPHS.

The value of an expression containing x undergoes changes when different values are assigned to x. It is often desirable to represent the changes undergone by means of a diagram, or **graph.** What is meant is best understood by examples.

Example 1.

Plot a number of points A, B, C, D, E, each of which has an abscissa of 5 units.

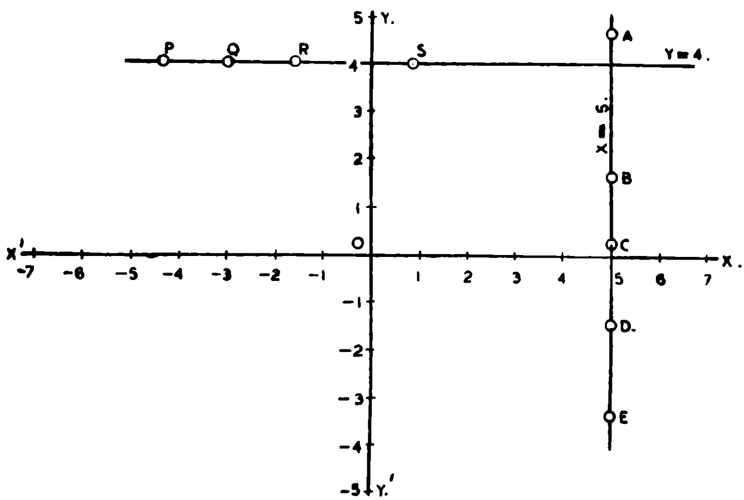


Fig. 2.

All these points lie on a straight line AE which is parallel to $Y'OY$ and 5 units distant from it. Any point not on this line AE, has an abscissa which is not equal to 5. Thus, the line AE contains all the points for which $x = 5$, and no other points. This line AE is called **the graph of $x = 5$.**

In the same way, if a number of points P, Q, R, S, are plotted, each with an ordinate of 4 units, it is found that they all lie on a straight line parallel to $X'OX$ and 4 units distant from it. All points not on this line PS do not have ordinates equal to 4. Hence **the line PS is the graph of $y = 4$.**

2. Graphs of Algebraic Expressions.—Straight Line Graphs—*continued.*

Example 2.

If the points (1, 1), (2, 2), (4, 4), (5, 5), (− 2, − 2), (− 4, − 4), are plotted, it is seen that they all lie on a straight line, which passes through the origin O. Also, if any point on this line is chosen, it is seen that its abscissa is equal to its ordinate, and that this is not true for any other point not on the line. Hence this line **is the graph of $x = y$ or $y = x$.**

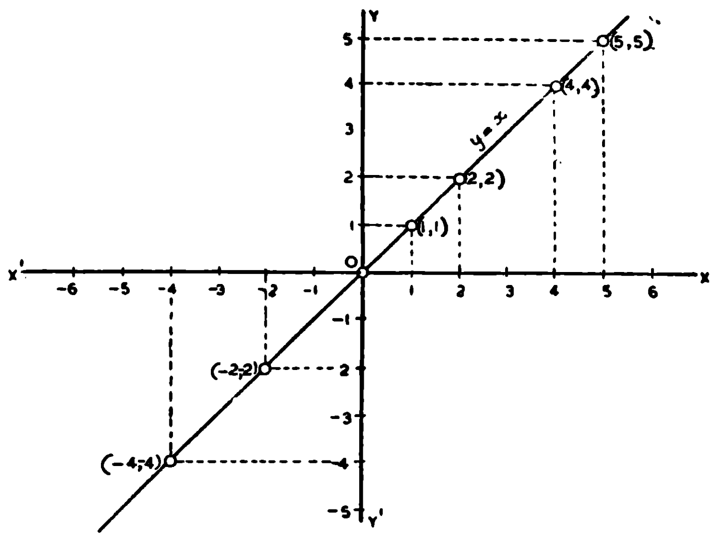


Fig. 3.

Example 3.

To draw the graph of $y = 2x$, a series of points is plotted, for each of which the ordinate is double the abscissa. By taking various values of x, and substituting them in the expression $2x$, the corresponding values of y are obtained. Thus :—

When $x =$	− 4	− 3	− 2	0	1	2	3	4
then $y =$	− 8	− 6	− 4	0	2	4	6	8

2. Graphs of Algebraic Expressions.—Straight Line Graphs—*continued.*

Now plot these points :—

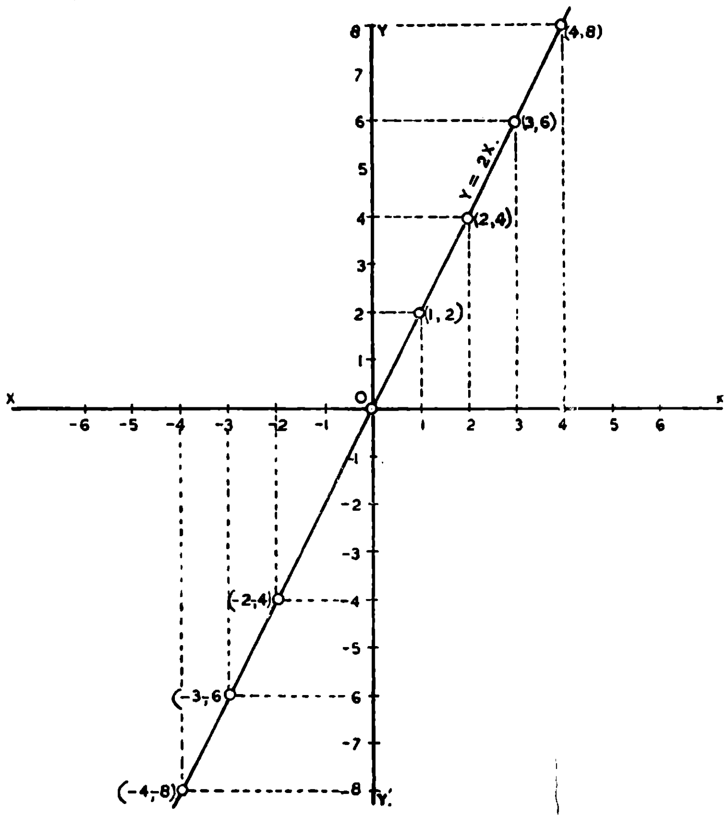


Fig. 4.

It is again seen that the points lie on
through the origin.

Example 4.

Consider the graph of $y = 2x - 3$, o
$2x - 3$. To obtain points, proceed as
convenient values for x and substitute
$2x - 3$, to find the corresponding values

Thus :—

When $x =$	-2	-1	0
then $y =$	-7	-5	-3

2. Graphs of Algebraic Expressions.—Straight Line Graphs—*continued.*

Example 4—*continued.*

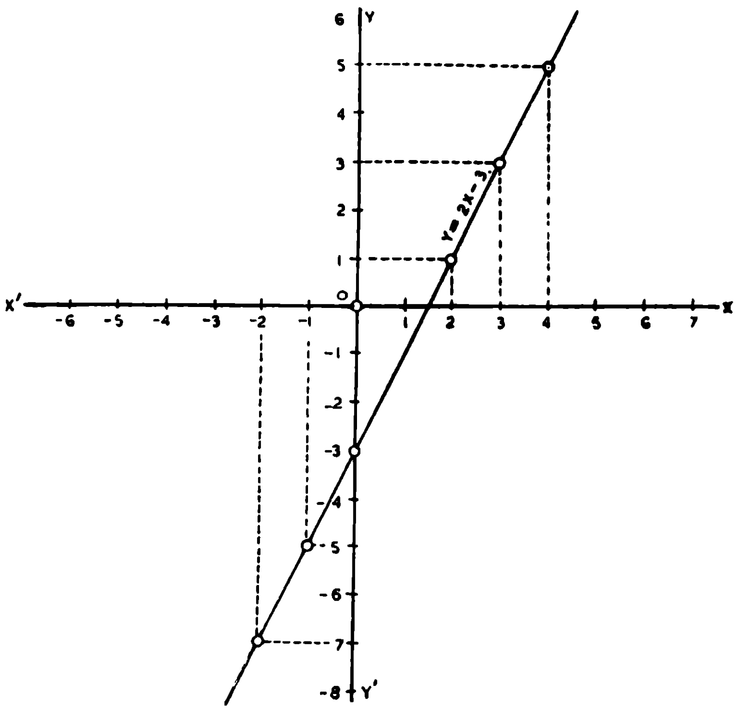


Fig. 5.

these points, it is again seen that they lie on a
his case it **does not pass through the origin.**

ıcts, relating to the graphs of the various
far, is as follows :—

ht line graphs.

a, where a is a constant, is a straight
the y-axis (e.g., $x = 5$, above).

b, where b is a constant, is a straight
the x-axis (e.g., $y = 4$, above).

mx where m is a constant, is a straight
through the origin (e.g., $y = x$, and
e). **N.B.** $m = 1$ in the first case, and
ond.

of $y = mx + c$, where m and c are constants,
ight line which does not pass through the
.g., $y = 2x - 3$ above). **N.B.** $m = 2, c = -3$.

Straight Line Graphs—*continued.*

4. In practice, graphs are always drawn on specially ruled graph paper, the usual interval between lines being $\frac{1}{10}$ inch.

In plotting graphs of expressions similar to those given above it is common to take five such intervals (i.e., $\frac{1}{2}$ in.) as the unit along each axis, though this may be varied if desired, as when x or y have fractional values. The unit used must always be clearly indicated on the paper, and the simplest and most useful way of doing this is to number the units along the axes, as shown in the above diagrams.

The resulting graph should be named by having the corresponding equation written clearly along some part of it, as has been done above. This is especially important when more than one graph is drawn on the same pair of axes.

So, in drawing graphs, remember to—

(1) Letter the axes.

(2) Show the scales chosen and number the units along the axes.

(3) Name the graph by writing alongside it the corresponding equation.

(4) Watch general neatness and use a sharp pencil.

EXERCISES.

1. Write down the co-ordinates of each of the points A, B, C, D, E, F, G, and H marked on the accompanying diagram.

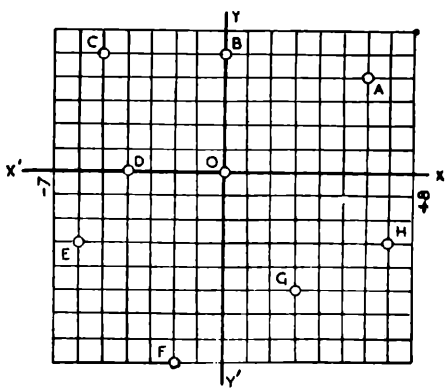


Fig. 6.

2. Using squared paper and $\frac{1}{2}$ inch as unit, plot the following points :—

$A = (5, 3)$; $B = (4, -6)$; $C = (-2, 5)$; $D = (-3, -1)$; $E = (3, 0)$; $F = (0, -4)$; $G = (-5, -5)$; $H = (6, -3)$.

3. On the same pair of axes, draw the graphs of the following :—

(i) $x = 4$, (ii) $y = 5$, (iii) $x = -2$, (iv) $y = -3$.

Indicate the units clearly, and label each graph.

Exercises—*continued.*

4. On a second graph **sheet,** draw the graphs of the following:
(i) $y = 3x$, (ii) $y = -2x$, (iii) $y = \frac{1}{2}x$.

Indicate the units **clearly** and label each graph.

5. Draw the **graphs of** the following equations. Show **on the** paper all the working in obtaining the necessary co-ordinates. All the graphs may be drawn, using the one pair of axes, provided that each graph and each set of working is clearly labelled.

(*a*) $y = 2x + 1$.
(*b*) $y = 5 - 6x$.
(*c*) $3x + 4y = 0$.
(*d*) $\frac{x}{2} - \frac{y}{3} = 1$.

REVISION.

1. USE OF FORMULAE.

(i) Substitute values of known symbols to find that of unknown.

(ii) Subject may be changed by multiplication, division or transposition.

2. CO-ORDINATES.

(*a*) The **axes** are the central horizontal and vertical lines.
($X'OX$ and $Y'OY$ respectively.)

(*b*) The **origin** is the point of intersection of the axes.

(*c*) Measurements to the **right** of the $Y'OY$ axis are positive, those to the **left** of it negative.

Measurements **upwards** from the $X'OX$ axis are positive, and those **downwards** from it negative.

(*d*) The **position of a point** is fixed by its perpendicular distances from the two axes. These distances are called the **co-ordinates** of the point.

(*e*) The **abscissa** is the horizontal distance of a point from the $Y'OY$ axis. It is denoted by x, and is **always given first.**

(*f*) The **ordinate** is the vertical distance of a point above or below the $X'OX$ axis. It is denoted by y, and is **always given second.**

3. STRAIGHT-LINE GRAPHS.

(*a*) Graphs of equations of the first degree (i.e., **containing** no squares) are all straight lines.

(*b*) Make y the subject of the **equation.**

(*c*) Substitute **three** convenient **values** of x and **find corresponding** values of y.

(*d*) Plot two of these three points and join **them.** As a check of the work performed, the third point should lie on the graph.

MISCELLANEOUS EXERCISES.

1. A cask is $\frac{7}{8}$ full. After 36 gallons have been drawn off, the cask is $\frac{1}{2}$ full. How many gallons does the cask contain when full ?

2. A man has a certain number of sovereigns, and twice as many half crowns. If he has £10 in all, how many of each coin has he ?

3. In a village consisting of 151 persons, there are 17 more women than men, and 30 more children than women. How many men, women, and children are there ?

4. A man walks a certain distance, cycles twice that distance and runs half as far as he walked. If he covers 14 miles altogether, how far did he walk ?

5. If a room is L ft. long, B ft. wide, and H ft. high, the area of its walls is $2H\ (L + B)$. Find the area of the walls of a room 10 ft. high, 12 ft. 6 in. wide, and 15 ft. long.

6. The area A of a circle is given by the formula $A = \pi r^2$, where r is the radius and $\pi = 3\frac{1}{7}$. Find the area of a circle whose radius is $5\frac{1}{4}$ in.

7. Rearrange the formula of No. 6 so as to make r the subject, and then find the radius of a circle whose area is 154 sq. in.

8. If a body falls freely under gravity, the distance s in feet is given by the formula $s = \frac{1}{2}gt^2$, where t is the time in seconds, and $g = 32$ feet per sec^2. A stone is dropped from the top of a cliff into the sea, and strikes the water $4\frac{1}{2}$ seconds later. How high is the cliff ?

9. The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base, h is the height, and $\pi = 3\frac{1}{7}$. Rearrange the formula so as to make h the subject; and then find the height of a cone whose volume is 2310 cub. in. and whose base has a radius of $10\frac{1}{2}$ in.

10. A merchant has two grades of oil worth 10s. and 16s. per gallon; how many gallons of each must he take to make a mixture of 40 gallons worth 11s. 6d. per gallon ?

11. In order that an aircraft of weight W lb. may fly at a velocity of V miles per hour, the area of its wing surface must satisfy the equation $\frac{W}{A} = KV^2$. Calculate A to nearest whole number if $K = 0{\cdot}0027$, $V = 45$ and $W = 2000$.

12. Approximate values for the length D (in statute miles) of a degree of longitude at various latitudes up to 60° can be obtained from the formula :—

$$D = 69 - \frac{x^2}{100}, \text{ where } x = \text{latitude in degrees.}$$

Use this formula to calculate—

(a) the length of a degree of longitude at London, latitude $51\frac{1}{2}°$, and

(b) the latitude at which $D = 62\frac{3}{4}$ miles.

Miscellaneous Exercises—*continued.*

13. Electrically-warmed flying suits, supplied with power at the rate of W watts, provide heat at the rate of 14 W units per minute. If W = voltage V (in volts) multiplied by current I (in amperes), find the current required to furnish a suit with 3,360 heat units per minute, the supply voltage being 24 volts.

14. The following table gives the specific resistance of the acid in accumulator batteries for different specific gravities. Plot a curve showing resistance plotted against specific gravity.

Read off the specific gravity at which the resistance is least and also the resistance at that point.

Specific Gravity..	1·04	1·06	1·09	1·1	1·15	1·2	1·25	1·3	1·35	1·4
Specific Resistance	2·5	2·0	1·9	1·5	1·3	1·3	1·4	1·7	2·15	2·9

15. The sag S (in inches) which occurs in a wire L feet long when stretched by a tension of T lb., depends largely on the weight W of the wire. If W is measured in ounces per foot then :—

$$S = \frac{3L^2W}{32T}.$$

Find the length of a wire, weighing 3 oz. per foot length, which sags by one foot when under a horizontal tension of 150 lb.

PRELIMINARY COURSE

ALL CADETS

Geometry 1C/1

GEOMETRICAL INSTRUMENTS REQUIRED.

To perform the exercises in the following short course in Geometry, you should provide yourself with :—

(i) A scale, one edge being graduated in centimetres and millimetres, and the other in inches and tenths of an inch.

(ii) Two set squares, one with angles of 45° and the other with angles of 60° and 30°.

(iii) A pair of pencil compasses.

(iv) A pair of dividers.

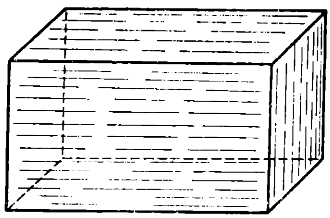
(v) A circular transparent protractor, i.e., one reading 360°. (Scale marked both clockwise and anti-clockwise.)

(vi) A sheet of transparent tracing paper.

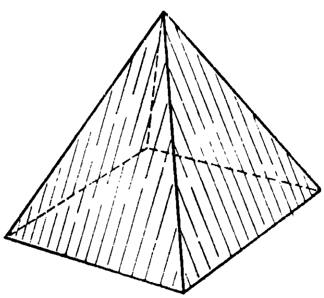
(Do not use a good scale for ruling lines. Use a set-square or a straight-edge kept for the purpose.)

1. FUNDAMENTAL TERMS AND DEFINITIONS.

Take a solid of the shape shown in the diagram. Notice that there are six faces to this solid. Each face meets four other faces in four straight edges. Three edges meet at each of the eight corners. We have used the terms, "faces," "edges," and "corners." In Geometry we should speak of a face as a **surface** (a **plane** surface in this case, because it is **flat**), the edges as **straight lines** and the corners as **points**.



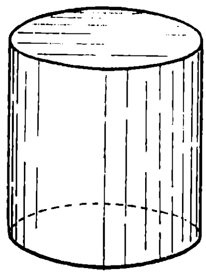
Before proceeding to give the strict geometrical definitions of the terms, surfaces, lines and points, let us examine some other common solids.



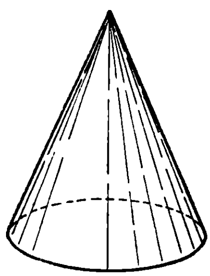
1. Fundamental Terms and Definitions—*continued.*

Consider the solid called **a pyramid.** The pyramid illustrated is of the form of those occurring in Egypt. Notice that in this case there are five surfaces (or faces), each meeting those adjacent to it in straight lines (or edges). Each straight line meets others in points (or corners).

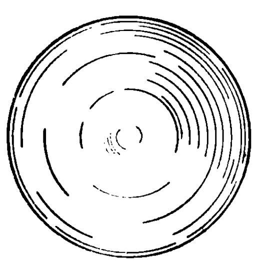
The next figure illustrated is **a cylinder.** This time we should notice some differences. There are three surfaces but only two are plane surfaces, the top end and the bottom. The third is **a curved surface.** The surfaces do not meet each other in straight lines but in **curved lines.**



A cone will be seen to possess two surfaces only, one of which is plane and the other curved. The two surfaces meet in one continuous curved line.



Finally, it will be seen that a **sphere** possesses only one surface and this is curved.



2. POINTS.

In Geometry, we regard a point as denoting position only, and as possessing no size. We think only of its location. A dot made by a sharp pencil is used to represent a point, but it should be realized that the smaller we make the dot the nearer will it approximate a geometrical point. Points are denoted by letters; thus we may speak of "point A" and "point B."

3. LINES.

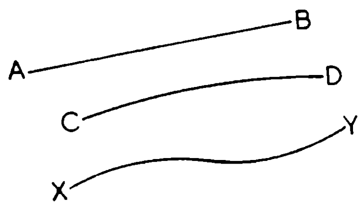
A line is the path traced out by a moving point. If the tip of a sharp pencil is placed upon a sheet of paper and drawn across it, the moving tip traces a mark which represents a line.

All lines have **length** but no breadth. The finer the mark traced by the pencil, the more closely it represents a line.

If a point moves continually in the same direction, its path is said to be **a straight line,** but, if it continually changes its direction, it traces out **a curved line.**

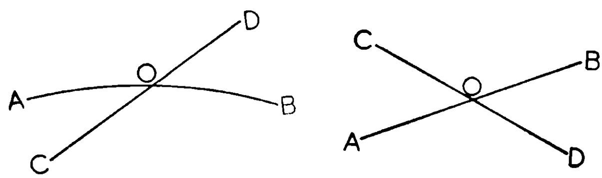
A line may be named by two letters placed at its ends.

Thus we speak of lines AB, CD, XY, &c.



The following are facts about lines that should be carefully noted :—

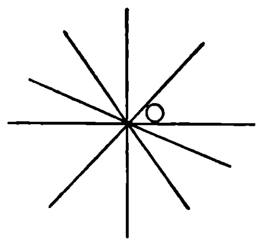
(A) Lines intersect (or cut) at a point, e.g., O in the diagrams.



(B) The shortest distance between two points is the length of the straight line joining them. Only one straight line can be drawn joining two points.

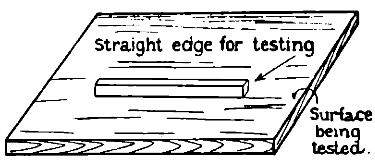
Lines—*continued.*

(C) Through any one point, however, any number of straight lines may be drawn.

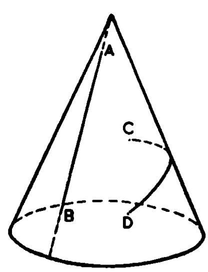


4. SURFACES.

A flat (or plane) surface is one in which the straight line joining any two points on it lies wholly within that surface. The surface of a piece of wood that has just been planed may be tested for flatness by taking a straight edge and placing it in various positions on the surface. If there are any hollows in the wood, light will be seen between the surface and the straight edge when held up for inspection.



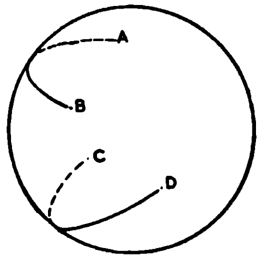
A straight line AB can be drawn on the curved surface of a cone by drawing it, in the manner shown, i.e., through its tip (or **vertex**) directly down the sloping surface. All lines drawn on the curved surface of the cone which do not pass through the vertex are curved lines.



Straight lines cannot be drawn upon the surface of a sphere. All lines drawn on its surface are curved. If the sphere were sufficiently large (say the size of the Earth) lines that may be

4. **Surfaces**—*continued.*

regarded as straight lines can be drawn on its surface. The earth very closely approximates a sphere but actually is slightly flattened at the poles.

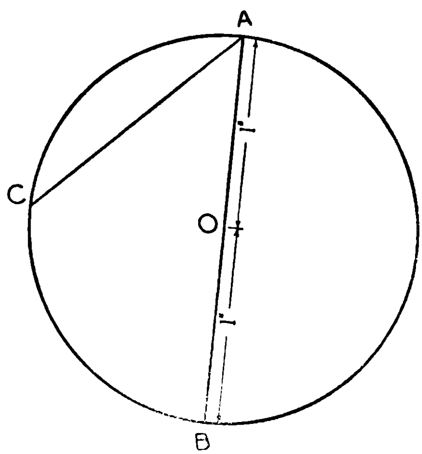


The surface of the page of your exercise book may be regarded as a plane surface.

5. THE MATHEMATICAL INSTRUMENTS.

(A) THE COMPASSES.

By using the scale, open the compasses to a distance of 1 inch. Select a convenient point *O* on a sheet of paper. Place the point of the compasses on it and draw a curved line with the pencil point. You will observe that the point of the pencil always keeps the same distance from *O* and, in tracing the line, the point of the pencil eventually comes back to he point from which it started (*A* in the diagram).



The figure thus drawn is **a circle.** It is said to be **a plane figure.** A plane figure is a portion of a plane surface enclosed by one or more lines.

The line traced by the pencil is called **the circumference** and *O* is called **the centre** of the circle. The length of the straight line drawn from the centre of a circle to any point on its circumference

5. The Mathematical Instruments—*continued.*

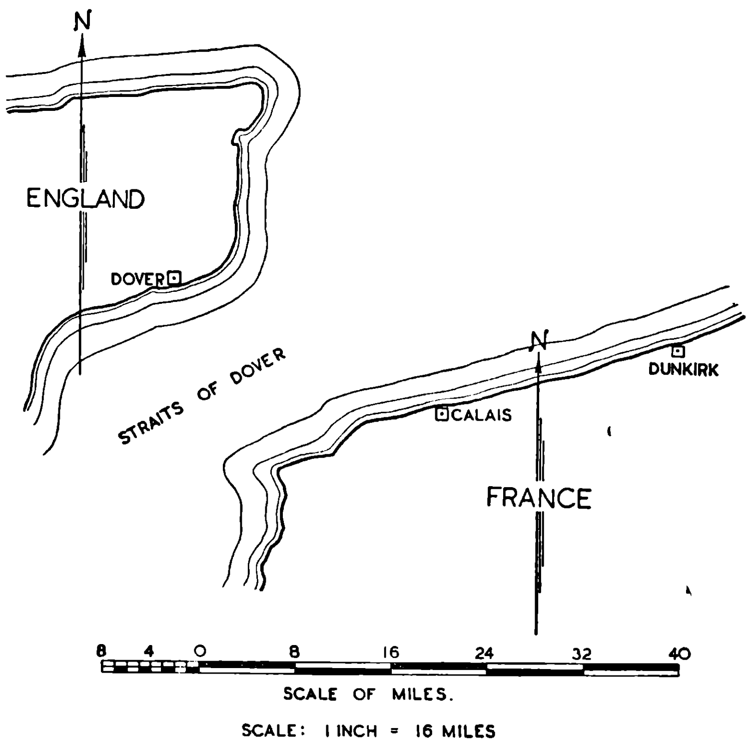
is called **the radius** of the circle. The distance OA is a radius of the circle drawn. All the radii of a circle are of the same length. (The plural of radius is radii.)

If the radius AO is produced to meet the circumference at B, the line AB is called **a diameter** of the circle. A diameter is thus a straight line drawn through the centre of a circle and terminated at each end by the circumference. Its length is twice that of the radius. Any diameter divides the circle into two equal parts, called **semi-circles.**

Suppose, starting from A, we proceed along the circumference of the circle to a point C. The portion AC of the circumference is called **an arc.** A straight line joining A and C, the extremities of the arc, is called **a chord.**

(B) THE DIVIDERS.

The accompanying diagram represents a portion of a map on which Dover, Calais and Dunkirk are marked. At the bottom of the map you will notice that a scale diagram has been constructed



5. The Mathematical Instruments—*continued.*

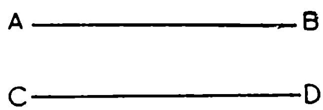
which shows the relation between distances on the map and distances as measured on the Earth's surface. The actual distance between any two places on the map is represented by the scale.

For this map, 1 inch represents 16 miles. Place one point of your dividers on the centre of the square representing Dover and open the dividers until the other point is on the centre of the square representing Calais. Transfer the dividers to the scale and find what distance the span of the dividers represents. Observe that sub-divisions of the 8-mile intervals are shown to the left of the zero position on the scale. Repeat the operation for the distance Dover to Dunkirk.

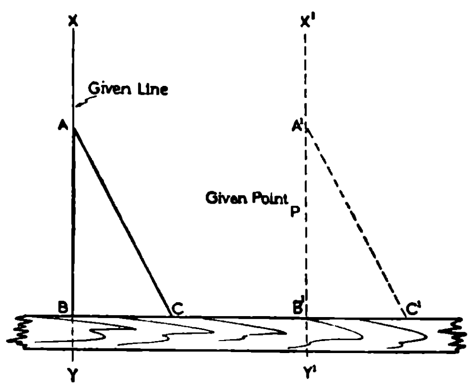
The answers should be :—25 miles, 42 miles.

(C) THE SET SQUARES AND STRAIGHT EDGE.

The set squares and a straight edge can be used for drawing through a given point, one straight line parallel to another given straight line. Two lines drawn on a plane surface are **parallel** when they do not meet, however far they are produced beyond both ends.



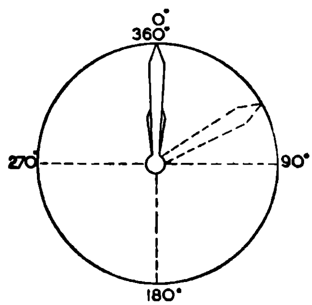
Suppose XY is a given straight line and P a given point through which we have to draw a straight line parallel to XY. Take a set square and place one edge AB along XY. Place the straight edge firmly against the edge CB as shown. Slide the set square along the straight edge, which must be firmly held, until the point P lies along the edge AB. A line $X'Y'$, drawn along the new position of the edge AB, and through P, will be the required line parallel to XY.



5. **The Mathematical Instruments**—*continued.*

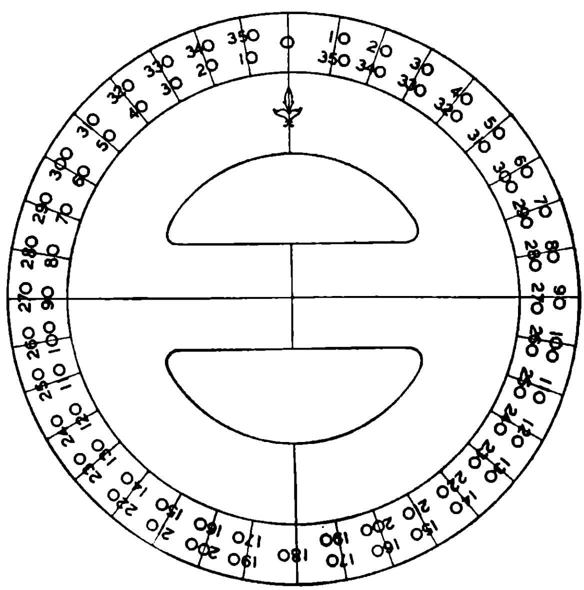
(D) THE PROTRACTOR.

Before actually using this instrument it is necessary that you should understand the meaning of the term, **angle.**



In the diagram two arms of different lengths, pivoted at the centre of the circle, are shown, like minute and hour hands of a clock. Consider them as being originally in the 12 o'clock position. If the "minute hand" is rotated it is said to trace out **an angle** with relation to the "hour hand."

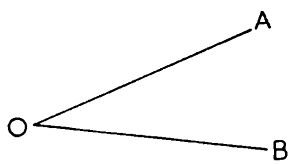
Angles are measured in **degrees.** If the minute hand made a complete revolution, i.e., right round the clock, it would trace out **360°**. One-quarter of a complete revolution consists of 90° and



5. **The Mathematical Instruments**—*continued.*

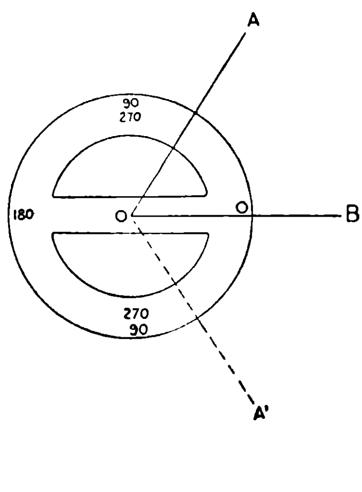
this is called **a right-angle.** An angle less than 90° is called **an acute angle;** one between 90° and 180° is called **an obtuse angle.** Angles from 0° to 360° can be read directly from your circular protractor. Most circular protractors have the scale marked both clockwise and anti-clockwise. It is desirable to have one so marked.

Angles are named by three letters, thus, from the accompanying diagram we should read "angle AOB." The point O is called the **vertex** of the angle and the lines OA and OB its **arms.** The abbreviation symbol for angle is $\angle$.



To measure the size of the angle AOB, **(or,** $\angle AOB$**),** place the protractor with its centre at O and the line marking the $0°$ position along OA. The value of the angle can then be read off on the protractor scale where OB (or OB produced) crosses it. Alternatively, the $0°$ position line could have been placed along OB and a reading made for where OA (or OA produced) crosses the protractor scale.

To draw an angle of a given number of degrees, say 57°. Draw one arm of the angle OB, centre the protractor on O and place the $0°$ position line along OB. The 57° can then be read off on the protractor scale on whichever side of the line the angle has to be drawn.



EXERCISES.

1. The floor of a rectangular room is 20 ft. long and 15 ft. wide. Draw a diagram of the floor, using a scale of 1 inch to 10 feet. From your diagram, find the actual distance between two opposite corners.

2. Draw a straight line AB, $2\frac{1}{2}''$ long. At A draw an angle BAC equal to $40°$, making AC equal to $2''$ in length.

Join C to B, thus completing the triangle ACB. Measure the length of CB and read off the values of the angles ACB, ABC.

Add together the values of the angles BAC, ACB, ABC. What do you notice about their sum ?

Produce the line AC to X. Measure the angles ACB, BCX and add their values together. What do you again notice ?

3. Draw two lines AB and CD intersecting at O. Measure each of the four angles so formed. What do you notice about the values of the angles, (*a*) AOC and BOD, (*b*) AOD and BOC ? Add the values of the four angles together. What can you say about their sum ?

4. Take a sheet of transparent tracing-paper and trace the map of the Dover–Calais area in this lesson. Transfer the tracing very carefully to the page of your book.

   (*a*) Show that the two North directional lines (or, **meridians** as they are called) are parallel on this map. (**But note carefully** :—On all maps they are not parallel, as will be learnt by those who proceed to the Navigation Course.)

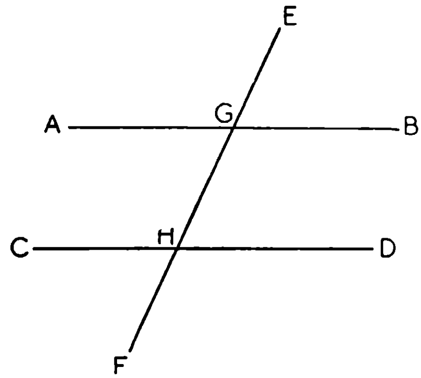
   (*b*) Through the centre of the square representing Dover draw a line parallel to either of the meridians.

   (*c*) Join the centre of the Dover square to the corresponding point for Calais. Measure the angle this line makes with the meridian already drawn through the position of Dover, the degrees being read from the meridian in a clockwise direction.

   (*d*) Through the centre of the square marking Dunkirk, draw a line parallel to the meridians. Join the position of Dunkirk to the centre of the square marking Dover. Measure the angle this line makes with the meridian of Dover, the degrees being read, as before, in a clockwise direction. Read off the angle between the two lines Dover-Dunkirk, Dover-Calais.

Exercises—*continued.*

5. Draw two parallel straight lines, AB and CD. Draw any straight line EF cutting them both in G and H respectively. Measure the angles in pairs as indicated below. Write down their values.



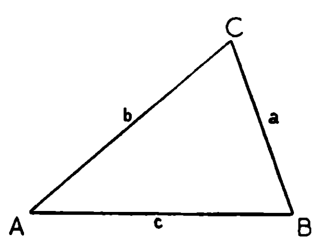
(*a*) $\angle$'s AGH and GHD, (*b*) $\angle$'s BGH and GHC, (*c*) $\angle$'s AGE and GHC, (*d*) $\angle$'s EGB and GHD, (*e*) $\angle$'s CHF and AGH, (*f*) $\angle$'s FHD and HGB. Add together the values of the angles AGH and CHG, and the angles BGH and DHG.

State in each case what you observe.

NOTE.

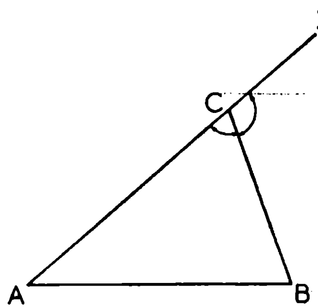
From the answers to your exercises you should have observed that in,

Question 2: (*a*) The three angles of the **triangle** ABC together equal 180° (or, 2 right angles). This holds for all triangles, various kinds of which will be considered in Lecture 2.

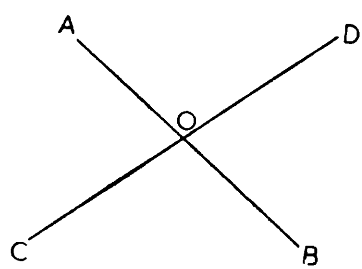


Exercises—*continued*

(*b*) The angles ACB and BCX together equal 180°. When one straight line BC meets another straight line AX, the **adjacent angles** so formed together equal two right angles.

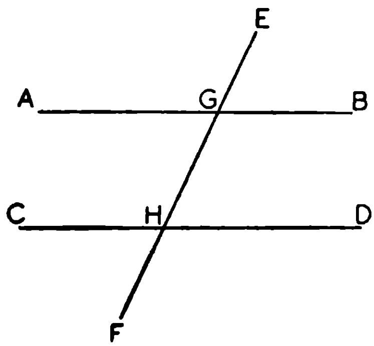


Question 3: The four angles formed by the intersection of two straight lines together equal 360° (or, 4 right angles). The **vertically opposite angles** AOC and DOB or AOD and COB are equal.



Exercises—*continued.*

Question 5: The **alternate** angles AGH and GHD are equal, as also are the alternate angles BGH and GHC.



The **exterior angle** AGE is equal to the interior opposite angle GHC. Similarly the exterior angles EGB, CHF and FHD are equal to the interior opposite angles GHD, AGH, and HGB respectively.

The **interior angles** AGH and CHG together equal 180° (or, 2 right angles). Likewise the interior angles BGH and DHG together equal 180°.

PRACTICAL PLANE GEOMETRY.

1. RECTILINEAL PLANE FIGURES.

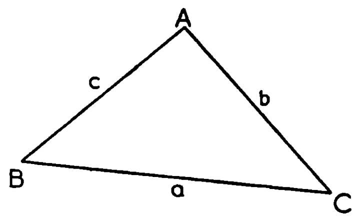
A Plane Figure is any area bounded by one or more lines lying in the same plane, i.e., it is portion of a plane surface.

The **Perimeter** is the sum of the bounding lines.

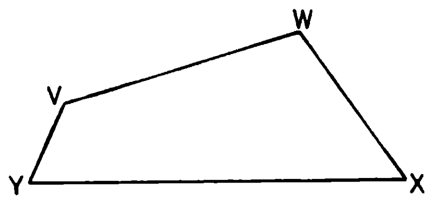
The **Area** is the amount of surface enclosed.

Rectilineal Figures are those bounded by **straight** lines and are classified into :—

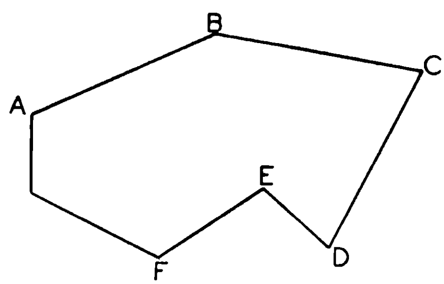
(A) **Triangles** which are bounded by three straight lines.



(B) **Quadrilaterals** which are bounded by four straight lines.



(C) **Polygons** which are bounded by more than four straight lines.

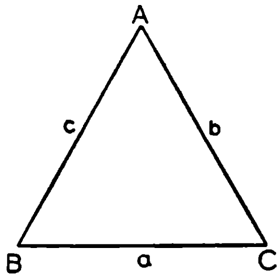


2. TRIANGLES.

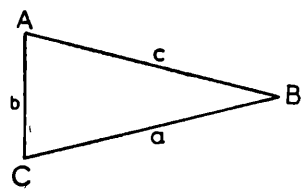
The six elements or parts of a triangle are its three sides and its three angles. In naming the parts of a triangle, capital letters are used for the angles and the corresponding small letters for the opposite sides (see sketches).

Triangles may be classified as follows :—

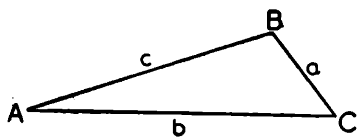
(A) **Equilateral**—all its sides equal. $a = b = c$.



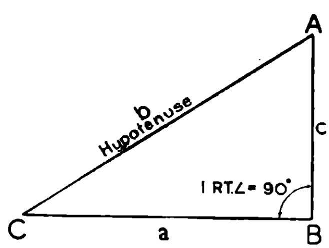
(B) **Isosceles**—two of its sides equal. $a = c$.



(C) **Scalene**—all its sides unequal.

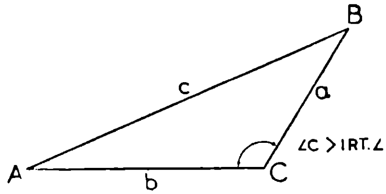


(D) **Right angled**—one angle is a right angle. The side opposite the right angle is called the **Hypotenuse.**

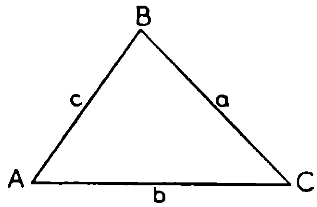


2. **Triangles**—*continued.*

(E) **Obtuse angled**—one angle is obtuse, i.e., greater than one right angle and less than 2 right angles.



(F) **Acute angled**—each of its angles is acute, i.e., less than one right angle.



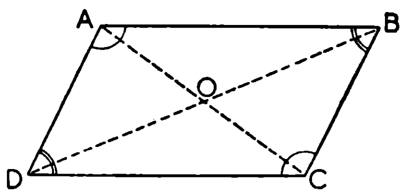
3. **QUADRILATERALS.**

The elements of a quadrilateral are its 4 sides and 4 angles.

NOTE : A straight line which joins opposite angles is called a **diagonal.** Quadrilaterals may be further classified as follows :—

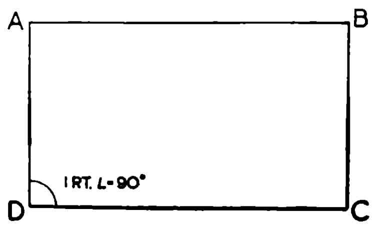
(A) **Parallelogram**—opposite sides are parallel. It follows that the opposite sides are equal; the opposite angles are equal; and the diagonals bisect each other.

$AB = DC$; $AD = BC$; $\angle A = \angle C$; $\angle D = \angle B$; $AO = OC$; $DO = OB$.



(B) **Rectangle**—a parallelogram with one angle a right angle. Hence all its angles are right angles.

$\angle A = \angle B = \angle C = \angle D = 1$ rt. $\angle = 90°$.



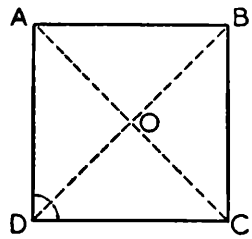
3. Quadrilaterals—*continued.*

(C) **Square**—a rectangle with 2 adjacent sides equal. In this case the diagonals bisect the angles as well as each other.

$\angle A = \angle B = \angle C = \angle D = 1$ rt. $\angle = 90°$.

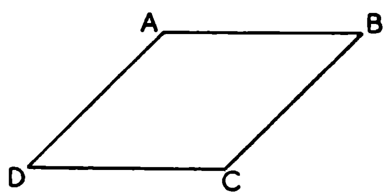
$AB = BC = CD = DA$.

$AO = OC = DO = OB$.



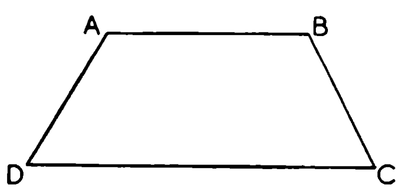
(D) **Rhombus**—a parallelogram which has all its sides equal, but its angles not right angles.

$AB = BC = CD = DA$.



(E) **Trapezium**—a quadrilateral which has one pair of parallel sides.

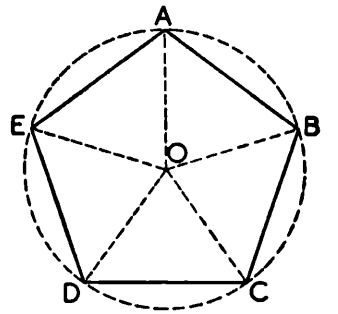
AB is parallel to DC.



4. POLYGONS.

In any **regular polygon** the sides are all equal and the angles formed by the intersection of the sides are equal. Further, the angles subtended by the sides at the centre of an inscribed or circumscribed circle, are all equal.

$$AB = BC = CD = DE = EA.$$
$$\angle A = \angle B = \angle C = \angle D = \angle E.$$
$$\angle AOE = \angle AOB = \angle BOC, \text{ \&c.}$$



5. ELEMENTARY GEOMETRICAL CONSTRUCTIONS.

The following are simple geometrical constructions, which are essential for construction of graphical diagrams.

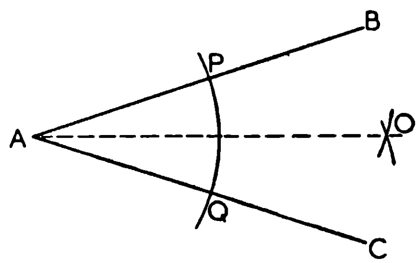
NOTE : Only a compass and straight edge are necessary for their construction, and hence errors due to set squares are eliminated.

(A) To bisect a given angle BAC.

With centre A and some convenient radius AP, draw an arc PQ, cutting AB and AC at P and Q.

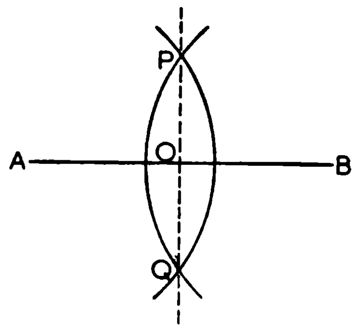
With centres P and Q and radius approximately equal PQ, draw two arcs intersecting at O.

Join AO, which bisects angle BAC, i.e., $\angle BAO = \angle OAC$.



5. Elementary Geometrical Constructions—*continued.*

(B) To bisect a given straight line AB.



With centres A and B, and radius AP greater than half AB, draw 2 arcs, which intersect at P and Q.

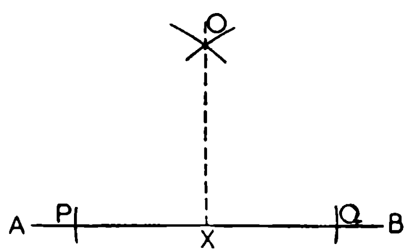
Join PQ, cutting AB at O.

Then AB is bisected at O.

Also PQ and AB are perpendicular to each other.

$AO = OB$; $\angle AOP = \angle POB = 1$ Rt. $\angle$.

(C) To draw a straight line perpendicular to a given straight line from a point X in it.

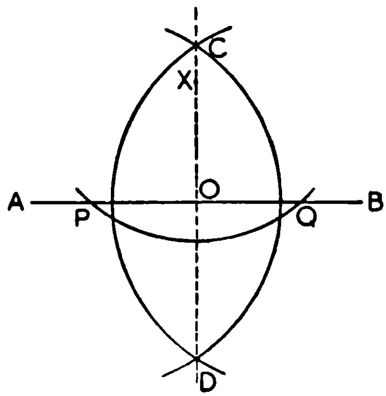


With centre X, cut off from AB two equal parts XP and XQ (use compass). With centres P and Q and radius approx. PQ, draw two arcs cutting at O.

Join XO, then XO is perpendicular to AB.

5. Elementary Geometrical Constructions—*continued.*

(D) **To draw a straight line perpendicular to a given straight line from a given external point X.**

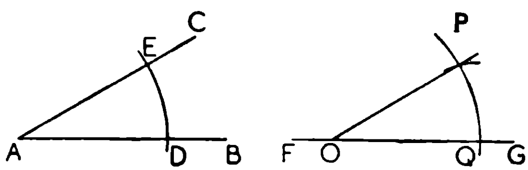


With centre X and some convenient radius draw arc cutting AB at points P and Q.

With centres P and Q and a radius approx. PQ, draw two arcs intersecting at C and D.

Join CD. Line CD passes through X and cuts AB at Rt. $\angle$'s at O.

(E) **At a given point O in a straight line FG, to draw an angle POQ equal to a given angle BAC.**



With centre A and any radius, draw arc DE cutting AB and AC.

With centre O and the same radius draw arc PQ, cutting FG at Q.

With centre Q and radius DE draw arc cutting first arc at P.

Join OP. Then POQ is required angle.

5. Elementary Geometrical Constructions—*continued.*

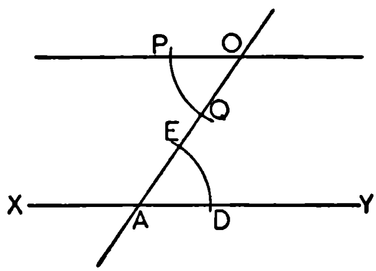
(F) **Through a given point** O **to draw a straight line** PO **parallel to a given straight line** XY.

From any point A in XY draw line AO. Then by construction given in (E) above, make angle POQ equal to angle OAY.

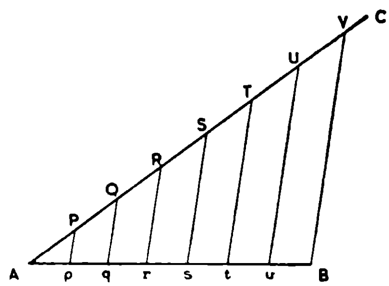
$\angle$'s POQ and OAY are alternate and equal.

Then OP is parallel to XY.

(See Lecture 1.)



(G) **To divide a straight line into any number of equal parts**



Method.

Let AB be the line to be divided into, say, 7 equal parts.

Draw AC at some angle to AB. From A mark off 7 equal parts of any convenient length, AP, PQ, &c.

Join VB and from points P, Q, R, &c., draw lines parallel to VB, cutting AB at p, q, r, &c.

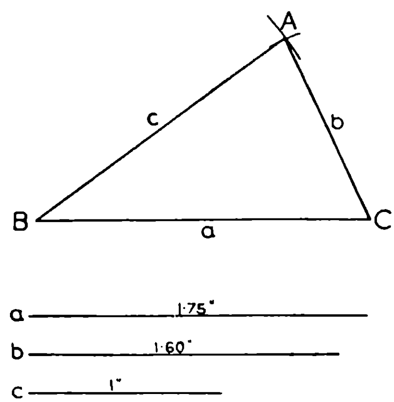
Then Ap, pq, qr, &c., are the required equal divisions of AB.

6. CONSTRUCTION OF TRIANGLES.

Diagrams drawn to scale are often used to solve problems in geometry, trigonometry, mechanics, electricity, &c. Most of these problems involve the construction of triangles from data that is given and finding the parts of the triangle that are not given.

To construct a triangle three of its parts must be given. The unknown parts can then be measured from the diagram.

(A) TO CONSTRUCT A TRIANGLE GIVEN THE LENGTHS OF THE THREE SIDES (i.e., *a*, *b*, and *c*).



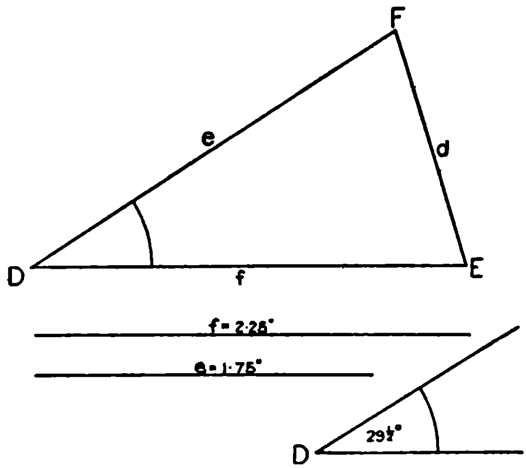
Method.

Let the lengths of the three sides *a*, *b* and *c* be as given in sketch (i.e., $a = 1{\cdot}75''$; $b = 1{\cdot}6''$; $c = 1{\cdot}0''$). Draw *BC* equal in length to *a*. With centre *B* and radius equal to *c* describe an arc as shown. With centre C and radius equal to *b*, describe an arc cutting the first arc at *A*. Join *BA* and *CA*. Then *ABC* is the required triangle and $\angle$'s *A*, *B* and *C* can be found by measurement.

6. Construction of Triangles—*continued.*

(B) TO CONSTRUCT A TRIANGLE GIVEN TWO SIDES AND THE INCLUDED ANGLE.

Let the given sides be f and e and the included angle D and have the values given in sketch.

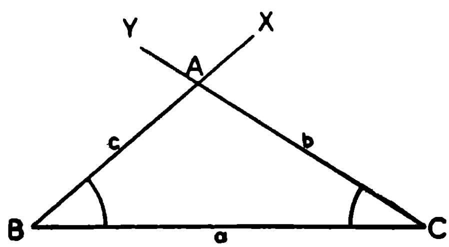


Method.

Draw DE equal in length to f. At D construct an angle FDE equal to the given angle. On DF mark F so that $DF = e$, the other given side. Join FE. Then FDE is the required triangle and $\angle F$, $\angle E$ and d can be found by measurement.

6. **Construction of Triangles**—*continued.*

(C) TO CONSTRUCT A TRIANGLE GIVEN ONE SIDE AND THE ANGLES AT ITS EXTREMITIES.



a = 2"; B = 40°; C = 30°

Method.

Let the given side be a and the given angles B and C with values as shown in sketch.

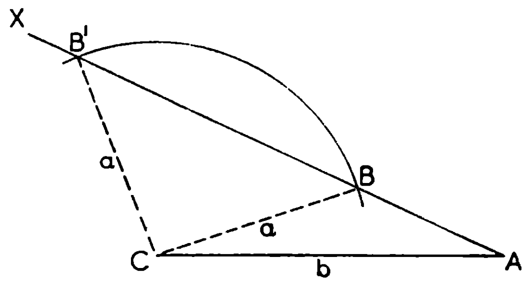
Draw BC equal in length to a. At B and C construct the $\angle$'s CBX and BCY equal to the given angles B and C respectively.

Then the point A at which BY and CY intersect is the third vertex of the required triangle and b, c, and $\angle A$ can be found by measurement.

If $\angle A$ and one of the $\angle$'s B or C had been given, the third angle could have been found by remembering that $\angle A + \angle B + \angle C = 180°$.

6. **Construction of Triangles**—*continued.*

(D) TO CONSTRUCT A TRIANGLE GIVEN TWO SIDES AND AN ANGLE OPPOSITE TO ONE OF THEM.



a = 1·1 ; b = 1·82" ; ∠A = 24°

Method.

Let the sides a and b and the angle A (opposite to a) be given with values as shown in sketch.

Draw AC equal to a. At A construct $\angle CAX$ equal to given $\angle A$. With centre C and radius equal to a describe an arc cutting AX in B and B'. Join BC and $B'C$. This gives two triangles:

(1) Obtuse angled triangle AB^1C.

(2) Acute angled triangle ABC.

Which both satisfy the given condition. This is known as the **Ambiguous Case.**

Note: (*a*) If a is greater than b then only one triangle can be constructed.

(*b*) If a is less than the perpendicular distance from C to AX then no triangle can be constructed.

EXERCISES.

1. Construct the following triangles:—

(*a*) $a = 3''$, $b = 3''$, $c = 3''$. Measure the three angles. What kind of triangle has been formed ?

(*b*) $a = 5''$, $b = 4''$, $c = 3''$. Again measure all angles. What kind of triangle has been formed ?

(*c*) $a = 1''$, $c = 1''$, $\angle ABC = 90°$. Measure AC, estimating its length to the nearest hundredth of an inch. Measure the remaining two angles.

Exercises—*continued.*

1. Construct the following triangles—*continued.*

   (*d*) $a = 2''$, $b = 3''$, $c = 3''$. Measure all angles. What kind of triangle has been formed ?

   (*e*) $a = 3''$, $\angle ABC = 40°$, $\angle ACB = 57°$. Measure the remaining elements of the triangle. What kind of triangle was constructed ?

2. Again trace very carefully the map of the Dover–Calais area. Join the centres called, say, D and C, of the squares representing Dover and Calais. Through D draw a line parallel to the meridians. From D along this line mark off a distance $DB = \frac{1}{4}''$ in a direction south of Dover. With centre B and radius $1''$ swing an arc to cut DC at A. Measure the length of DA.

3. Draw a line AB, $4''$ long, horizontally. At A draw an angle $BAD = 40°$ so that $AD = 2''$. Through D draw a line parallel to AB and through B draw a line parallel to AD. If these two lines intersect at C, measure all angles formed by the sides of the figure. State which angles are equal. Measure DC and BC. To what sides are each of these equal ? What do you notice about the sum of the angles ADC and DAB, and the angles DCB and ABC ? Measure the lengths of AC and DB.

4. (*a*) Draw a line AB, $1\frac{1}{2}''$ long. Divide it into 9 equal parts.

   (*b*) Divide a line $\frac{9}{10}$ inch long into 10 equal parts.

TRAINING COURSE.

GROUND DUTIES CADETS.

STAGE 1.

Trigonometry. 1D/1b/1.

REVISION OF PRELIMINARY COURSE.

1. Revise Lectures 1 and 2, 1C/1 Geometry.

2. Revision will include **Fundamental Terms and Definitions, Mathematical Instruments, Angles, Angular Measurement and Construction of Triangles.**

REVISION EXERCISES.

1. (*a*) Construct a triangle, given $a = 5\frac{1}{2}''$, $b = 4''$, $c = 2\frac{1}{2}''$.

(*b*) By means of a protractor, measure each angle to the nearest degree.

(*c*) Find the sum of the three angles. What should it be ?

2. (*a*) Draw a circle of radius 2 inches. Mark out an arc *AB* which subtends an angle of 40 degrees at the centre. How many consecutive arcs of this magnitude can be obtained ?

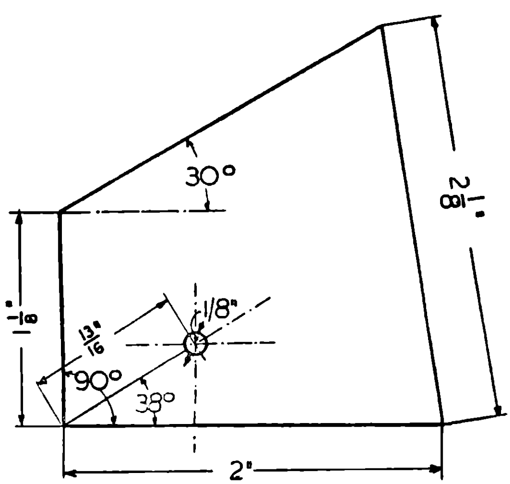
(*b*) Draw in the chord of one arc. Is the chord longer than the arc ? How would the length of arc be measured ?

3. A fitter is required to mark out the centres of 6 bolt holes equally spaced around the circumference of a circle $3\frac{1}{4}$ inches diameter. Perform the necessary construction.

4. On a plan, the rim of a steering wheel scales $4\frac{1}{4}$ inches outside diameter and $3\frac{3}{4}$ inches inside diameter. The hub scales $\frac{3}{4}$-inch diameter. There are 10 spokes equally spaced. Draw down the wheel to the dimensions given and mark in the spokes.

5. Construct a right-angled triangle with one side (not the Hypotenuse) = 3 inches, and one angle 25°. Measure the other two sides, and remaining angle. What value should the latter have ?

6. Set out the following plate as accurately as possible to full size.



How could the $\frac{1}{8}$-inch hole be better located ?

TRIGONOMETRICAL RATIOS.

1. The three Fundamental Trigonometrical Ratios are :—
   (i) sine.
   (ii) cosine.
   (iii) tangent.

2. CONSTRUCTION TO FIND RATIOS.

Consider an acute angle A (Fig. 1).

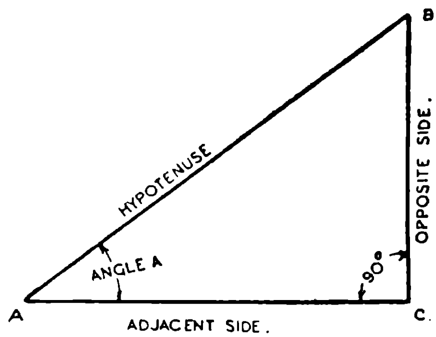


Fig. 1.

Take **any** point B in one of the arms and drop the perpendicular BC on to the other arm, forming the right-angled triangle ABC.

Then :—

(i) the side BC **opposite** to the angle A is called the **Opposite Side.**

(ii) the side AC **adjacent** to the angle A is called the **Adjacent Side.**

(iii) the side AB opposite to the right angle C is called the **Hypotenuse** (see Preliminary Course, 1C/1, Elementary Geometry).

3. DEFINITION OF RATIOS.

The three Fundamental Ratios may then be expressed as follows :—

(i) sine of the angle A, written **sin A** $= \dfrac{\text{Opposite side}}{\text{Hypotenuse}}$

$$= \frac{BC}{AB}$$

(ii) cosine of the angle A, written **cos A** $= \dfrac{\text{Adjacent side}}{\text{Hypotenuse}}$

$$= \frac{AC}{AB}$$

3. Definition of Ratios—*continued.*

(iii) tangent of the angle A, written **tan A** $= \dfrac{\text{Opposite side}}{\text{Adjacent side}}$

$$= \frac{BC}{AC}$$

N.B.—The angle A can have arms of **any length,** and is **not limited** by the size of the right-angled triangle ABC chosen to define its Trigonometrical Ratios. Thus to describe the angle A as $\angle BAC$, may lead to the idea that its arms are limited to AB and AC, whereas AB and AC only result from the necessary construction for determining, as well as defining, the Trigonometrical Ratios.

The values of the ratios are independent of the lengths of the arms AB and AC chosen. This will be shown in the following example.

EXAMPLE.

Find, by construction and scaling, the sin, cos and tan of 37°. Choose 3 different lengths of arms and verify the statement that the ratios are independent of the lengths of arms.

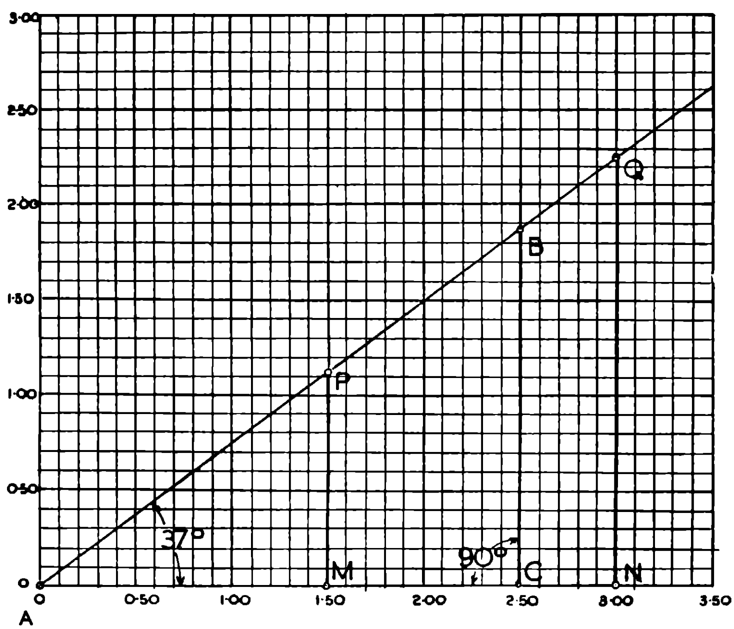


Fig. 2.

Draw a straight line, and at a point A on it, construct an angle of 37° (by protractor).

3. **Definition of Ratios**—*continued.*

Take **any** three points P, B and Q in one of the arms of the angle, and from these points draw PM, BC and QN **perpendicular** to the other arm.

Then, from Fig. 2 **by scaling** :—

$AP = 1{\cdot}87''$	$AM = 1{\cdot}50''$	$PM = 1{\cdot}12''$
$AB = 3{\cdot}13''$	$AC = 2{\cdot}50''$	$BC = 1{\cdot}87''$
$AQ = 3{\cdot}76''$	$AN = 3{\cdot}00''$	$QN = 2{\cdot}25''$

Ratios :—

$$\text{(i) } \sin A = \frac{PM}{AP} = \frac{1{\cdot}12}{1{\cdot}87} = 0{\cdot}598,\quad = \frac{BC}{AB} = \frac{1{\cdot}87}{3{\cdot}13} = 0{\cdot}597,\quad = \frac{QN}{AQ} = \frac{2{\cdot}25}{3{\cdot}76} = 0{\cdot}598$$

Average Value $= 0{\cdot}598$
From Tables of Natural Sines, $\sin 37° = 0{\cdot}602$

$$\text{(ii) } \cos A = \frac{AM}{AP} = \frac{1{\cdot}50}{1{\cdot}87} = 0{\cdot}802,\quad = \frac{AC}{AB} = \frac{2{\cdot}50}{3{\cdot}13} = 0{\cdot}799,\quad = \frac{AN}{AQ} = \frac{3{\cdot}00}{3{\cdot}76} = 0{\cdot}798$$

Average Value $= 0{\cdot}800$
From Tables of Natural Cosines, $\cos 37° = 0{\cdot}799$

$$\text{(iii) } \tan A = \frac{PM}{AM} = \frac{1{\cdot}12}{1{\cdot}50} = 0{\cdot}747,\quad = \frac{BC}{AC} = \frac{1{\cdot}87}{2{\cdot}50} = 0{\cdot}748,\quad = \frac{QN}{AN} = \frac{2{\cdot}25}{3{\cdot}00} = 0{\cdot}750$$

Average Value $= 0{\cdot}748$
From Tables of Natural Tangents, $\tan 37° = 0{\cdot}753$

N.B.—Accuracy in scaling will affect the constancy of **the ratios**. Best results are obtained with **large diagrams**. The use of **squared paper** is also desirable.

4. TRIGONOMETRICAL TABLES.

Graphical methods of finding the Trigonometrical Ratios are not convenient for use in most practical cases. Tables have, therefore, been prepared, from which values of the various Trigonometrical Ratios can be read off quickly and accurately.

4. Trigonometrical Tables—*continued.*

An extract from the Table of Natural Sines supplied will illustrate the general arrangement adopted in each of the respective tables.

NATURAL SINES

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences 1′	2′	3′	4′	5′
0°	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	9	12
36	·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11

NOTE.—(i) The ratios for each degree up to 90° are listed in the columns headed 0′ and can be read off **directly.** The ratios for angles containing minutes need not be considered.

(ii) The ratios are given to an accuracy of four significant figures.

(iii) Tables of Natural Sines, Cosines and Tangents are given in Appendix 1D/1b/1 Trigonometry.

EXERCISES.

1. Find, by construction and scaling, the sin, cos and tan of—
   (i) 25° (ii) 60°.

Verify, in each case, that the ratios are independent of the lengths of arms. Check results from the tables.

2. From the Tables of Natural Sines, find the value of—

(i) sin 10°	(iv) sin 45°	(vii) sin 69°
(ii) sin 25°	(v) sin 51°	(viii) sin 73°
(iii) sin 32°	(vi) sin 60°	(ix) sin 86°

3. From the Tables of Natural Cosines, find the value of—

(i) cos 5°	(iv) cos 37°	(vii) cos 75°
(ii) cos 14°	(v) cos 49°	(viii) cos 83°
(iii) cos 25°	(vi) cos 60°	(ix) cos 89°

4. From the Tables of Natural Tangents, find the value of—

(i) tan 2°	(iv) tan 34°	(vii) tan 60°
(ii) tan 17°	(v) tan 41°	(viii) tan 78°
(iii) tan 25°	(vi) tan 57°	(ix) tan 88°

5. What do you notice about the values of sin, cos, and tan as the angle increases from 0° to 90° ?

TRIGONOMETRICAL RATIOS—*continued.*

1. EASY PROBLEMS.

The type of problem most usually met with in practice will be the use of formulae involving trigonometrical ratios. The formulae will be **given** and do not require development. The angles involved are usually in **degrees,** and values of trigonometrical ratios can be read **directly** from the tables supplied. Practice in the use of formulae has already been obtained in Lecture 3, 1B/2b/1 Algebra.

The following examples will illustrate the procedure :—

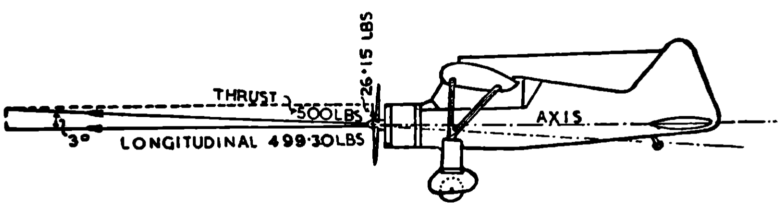
EXAMPLES.

(1) If the thrust of an aircraft's propeller is inclined at an angle A to the longitudinal axis of the aircraft, then the forces along and perpendicular to the longitudinal axis are given by the formulae :—

(i) Force along the axis $=$ (thrust) $\times$ ($\cos A$).

(ii) Force perpendicular to axis $=$ (thrust) $\times$ ($\sin A$).

If thrust $= 500$ lb. and angle $A = 3°$, find the forces along and perpendicular to the longitudinal axis.



(i) Force along the axis $=$ (thrust) $\times$ ($\cos A$)
$= 500 \times \cos 3°$ lb.
$= 500 \times 0{\cdot}9986$ lb.
$=$ **499·30 lb.**

(ii) Force perpendicular to axis $=$ (thrust) $\times$ ($\sin A$)
$= 500 \times \sin 3°$ lb.
$= 500 \times 0{\cdot}0523$ lb
$=$ **26·15 lb.**

(2) The **Total Reaction** on an aircraft is defined as the total force due to air flow on its wings. The **drag** is the resistance to motion acting in the direction of air flow, and the **lift** is the force tending to keep the aircraft in the air.

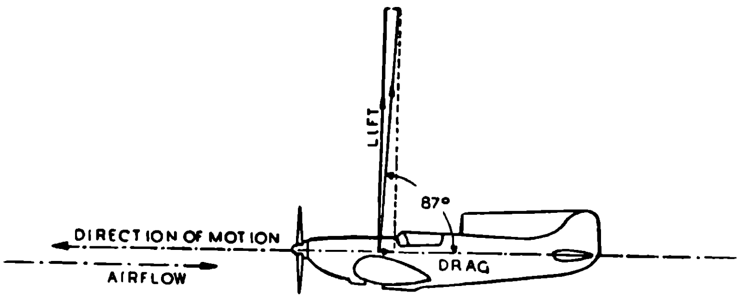
Examples—*continued.*

If the total reaction makes an angle A with the direction of air flow, then—

(i) lift $=$ (total reaction) $\times$ ($\sin A$).

(ii) drag $=$ (total reaction) $\times$ ($\cos A$).

Find, the lift and drag on an aircraft, given that the total reaction is $3 \cdot 4$ tons acting at $87°$ to the direction of air flow.



(i) Lift $= \text{(total reaction)} \times (\sin A)$
$= 3 \cdot 4 \times \sin 87° \text{ tons}$
$= 3 \cdot 4 \times 0 \cdot 9986 \text{ tons}$
$= \mathbf{3 \cdot 395 \text{ tons.}}$

(ii) Drag $= \text{(total reaction)} \times (\cos A)$
$= 3 \cdot 4 \times \cos 87°$
$= 3 \cdot 4 \times 0 \cdot 0523$
$= \mathbf{0 \cdot 178 \text{ tons.}}$

(3) If F, the total magnetic force at a point on the earth's surface, makes an angle A with the horizontal, then :—

(i) Horizontal magnetic force $= F \cos A$.

(ii) Vertical magnetic force $= F \sin A$.

At London the earth exerts a magnetic force F of $0 \cdot 47$ dyne, acting at $67°$ to the horizontal. Find the horizontal and vertical magnetic forces.

(i) Horizontal magnetic force $= F \cos A$
$= 0 \cdot 47 \times \cos 67° \text{ dyne}$
$= 0 \cdot 47 \times 0 \cdot 3907 \text{ dyne}$
$= \mathbf{0 \cdot 18 \text{ dyne.}}$

(ii) Vertical magnetic force $= F \sin A$
$= 0 \cdot 47 \times \sin 67° \text{ dyne}$
$= 0 \cdot 47 \times 0 \cdot 9205 \text{ dyne}$
$= \mathbf{0 \cdot 43 \text{ dyne.}}$

Examples—*continued.*

(4) In alternating current electrical work, the power absorbed by a single phase circuit is $VI \cos \theta$ watts, where V = voltage between lines; I = current in amperes; and $\cos \theta$ = the power factor of the circuit.

Find the power in watts, if $V = 210$ volts, $I = 5$ amperes, $\theta = 35°$.

$$\begin{aligned} \text{Power} &= VI \cos \theta \\ &= 210 \times 5 \times \cos 35° \text{ watts} \\ &= 210 \times 5 \times 0{\cdot}8192 \text{ watts} \\ &= 860{\cdot}2 \text{ watts.} \end{aligned}$$

EXERCISES.

N.B.—Draw a diagram for each exercise.

1. A model aeroplane is released from a height H, and travels on a straight path making an angle A with the vertical. If the horizontal distance D to its landing point is given by $D = H \tan A$, find D when $H = 25$ feet and $A = 75°$.

2. If W is the weight of a body which slides on a horizontal surface, and A is the angle of friction, then the frictional resistance to motion, R, is given by the formula:—

$$R = W \tan A.$$

Find R, when $W = 112$ lb., and A is (i) 3°, (ii) 11°, (iii) 16°, (iv) 24°. Give answers to the nearest tenth of a lb.

3. Find the force along and perpendicular to the axis of an aircraft, given that the thrust of 734 lb. makes an angle of 2° to the longitudinal axis.

4. In an aero-engine, if P is the force on the piston, and A the angle between the connecting rod and the centre line of the cylinder, then force R in the connecting rod is given by $R = P \cos A$. Find R, when $P = 2{,}500$ lb., and A is (i) 5°, (ii) 14°, (iii) 23°.

5. If the total reaction on a bomber is 15·3 tons acting at an angle of 86° with the direction of air flow, find the lift and drag.

6. Find the power developed in a single phase A.C. circuit, given the following:—

Voltage—V.	Amperage—I.	θ.	Power = $VI \cos \theta$ watts.
400	3·2	25°	
210	4·8	40°	
220	6·0	32°	
110	10·0	29°	
440	1·9	38°	

Trigonometrical Ratios—*continued.*

7. Find the horizontal and vertical magnetic forces at a point on the earth's surface, where the total magnetic force $F = 0 \cdot 41$ dyne, making an angle of 34° with the horizontal.

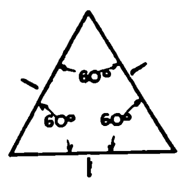
8. The force tending to bring a compass needle into the magnetic meridian is given as $F = 1 \cdot 76 \sin A$, where A is the angle between the axis of the needle and the meridian.

Find F, when A has the following values:—(i) 0°, (ii) 3°, (iii) 10°, (iv) 30°, (v) 45°, (vi) 60°, (vii) 75°, (viii) 90°.

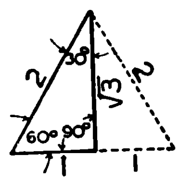
REVISION.

1. Angles are generally measured by a **protractor**; common angles such as 30°, 60°, 45° can be found by inspection from geometrical proportions. These should be known and readily recognized, viz.:—

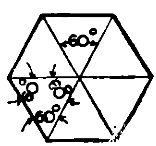
(i) **Equilateral** Triangle—each angle is 60°.



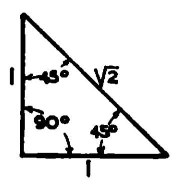
(ii) **Bisect one angle** of equilateral triangle to get 30°.



(iii) Sides of a **Hexagon** subtend angles of 60° at the centre.



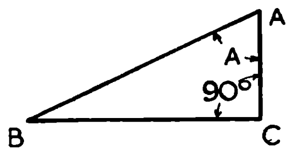
(iv) A **right-angled isosceles triangle** has angles of 45°.



2. Work with **right-angled** triangles by dropping perpendiculars.

Revision—*continued.*

3. From right-angled $\triangle$ ABC,



$$\sin A = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{CB}{AB}$$

$$\cos A = \frac{\text{Adjacent side}}{\text{Hypotenuse,}} = \frac{AC}{AB}$$

$$\tan A = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{CB}{AC}$$

4. Values of the Trigonometrical Ratios are independent of the length of arms AB and AC of the angle A.

5. Values of Ratios are **read off directly** from Tables of Natural Sines, Cosines and Tangents.

6. Only **angles in degrees** will be considered.

7. Problems met with in practice will be solution of formulae involving Trigonometrical Ratios.

REVISION EXERCISES.

1. Construct the plan (full size) of a hexagon nut given the following proportions:—

Width across corners $= 2 \times$ diameter of bolt.
Diameter of bolt $= 1\frac{1}{2}''$.

Find by measurement and calculation the width across flats.

2. Using a protractor, draw a triangle from the following dimensions—

one side $= 3\frac{1}{2}$ inches.
angles adjacent to given side $= 39°$ and $51°$.

Find, by construction and measurement, the sin, cos and tan of 39° and 51°. Check your results with the values from Tables.

3. The crank of an aero-engine has a radius of 5·3 inches. Find the perpendicular distance from the centre of the crank pin to a horizontal plane through the centre line of the crankshaft, when the crank makes angles of 15°, 21°, 48°, 60°, and 82° with the vertical. Draw a suitable diagram.

4. A searchlight at O catches an aircraft A in its beam when the beam is inclined at an angle of 56° with the horizontal. At the same time the range OA is observed by an instrument to be 3,500 yards. Calculate (1) the vertical height of the aircraft and, (2) the horizontal distance from O to a point vertically below A. Draw a diagram.

5. The connecting rod of an aero-engine is 10 inches long, and when the crank and connecting rod are at right angles, the connecting rod makes an angle of 26° with the centre line of the cylinder.

Find—

(i) the crank radius.
(ii) the distance between centres of crank pin and gudgeon pin in this position.

TABLES

OF

TRIGONOMETRICAL RATIOS.

Appendix 1D/1b/1 Trigonometry.

NATURAL SINES

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences 1′	2′	3′	4′	5′
0°	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	14
5	·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	3	6	9	11	14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	13
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	9	12
36	·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences				
											1′	2′	3′	4′	5′
45°	·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	·7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	·9998	9999	9999	9999	9999	0000	0000	0000	0000	0000	0	0	0	0	0

NATURAL COSINES

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	64′	Mean Differences 1′	2′	3′	4′	5′
0°	1·0000	0000	0000	0000	0000	0000	$\overline{9999}$	$\overline{9999}$	$\overline{9999}$	$\overline{9999}$	0	0	0	0	0
1	·9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	·9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	·9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	·9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	·9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	·9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	·9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	·9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	·9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	·9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	·9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	·9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	·9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	·9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	·9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	·9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	·9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
18	·9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	·9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	·9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	·9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	·9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	·9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
24	·9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	·9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	·8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	·8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	·8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	·8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	·8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	·8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	·8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	·8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	·8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	·8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	·8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
37	·7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	·7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	·7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	·7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	·7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	·7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	·7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	·7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

NATURAL COSINES

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences 1′	2′	3′	4′	5′
45°	·7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	·6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	·6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	·6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	·6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50	·6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	·6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	·6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	·6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	·5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	·5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	·5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	·5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	·5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	·5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	·5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
61	·4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	·4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	·4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	·4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	·4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	·4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	13
67	·3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	13
68	·3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	·3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	·3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	·3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	·3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	·2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	·2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	·2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	·2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	·2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	·1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
80	·1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	·1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	·1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
83	·1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	·1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	·0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	14
86	·0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	·0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	·0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	·0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15

NATURAL TANGENTS

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences				
											1′	2′	3′	4′	5′
0°	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences 1′	2′	3′	4′	5′
45°	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	0057	0145	0233	0323	0413	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2·9042	9208	9375	9544	9714	9887	0061	0237	0415	0595	29	58	87	116	145
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972					
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4·7046	7453	7867	8288	8716	9152	9594	0045	0504	0970					
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5·6713	7297	7894	8502	9124	9758	0405	1066	1742	2432	Mean differences				
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	0264	no longer				
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	0285	sufficiently				
83	8·1443	2636	3863	5126	6427	7769	9152	0579	2052	3572	accurate				
84	9·514	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95	Use Principle of				
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46	Proportional				
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27	Parts				
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0					

PRELIMINARY COURSE

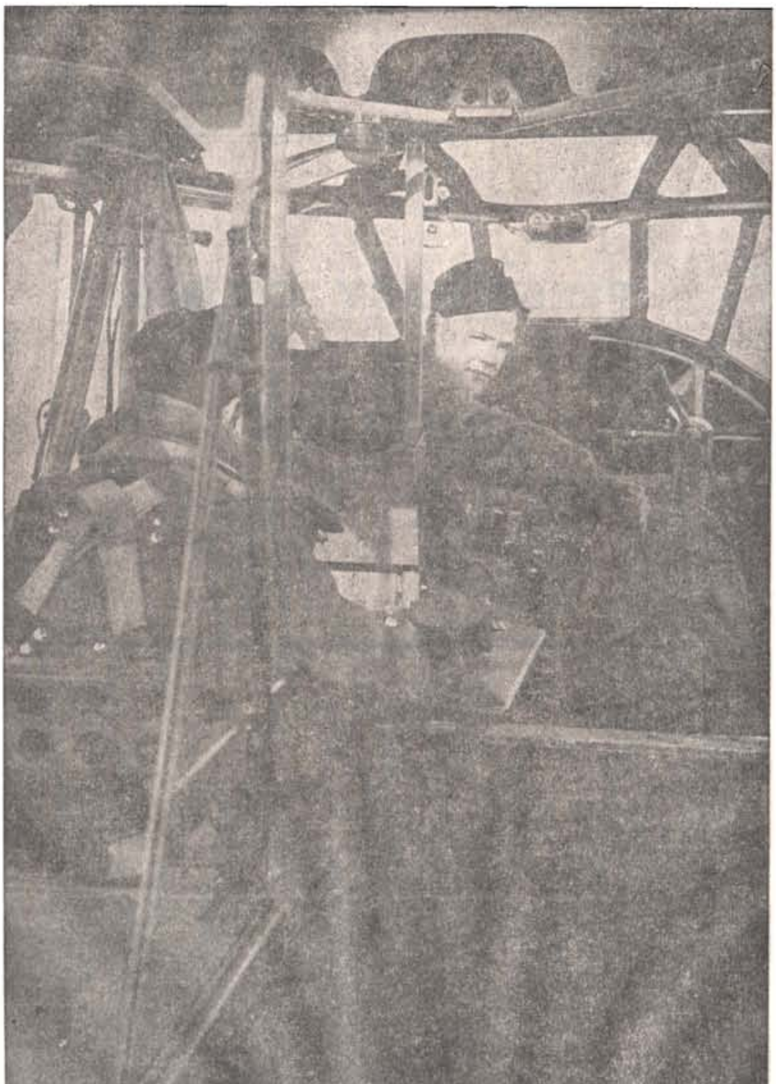
ALL CADETS

Science 2/1

MEASUREMENT.

1. INTRODUCTION.

Careful designing and great precision of workmanship have produced, for Britain, aircraft of the quality of the Spitfire. Accurate plotting by navigators has brought our long-range bombers safely back to their bases after extensive operations. Fundamentally the



A Navigator at work in the air.

1. Introduction—*continued.*

calculations involved in these achievements presuppose a clear appreciation of measurement. In fact, precision in measurement is essential at every stage in the building up and operation of an Air Force.

Plotting the course before the flight.

Measurement is not, however, entirely a product of modern times. Civilizations long past had methods of making measurements with surprising accuracy, and many of their observations have been handed down to us. From very early times Man felt both the need for aids to his senses and the means of expressing his observations and activities in a form capable of record. Not all his efforts met with the success we enjoy at present.

Units of measurement that are capable of variation are obviously highly undesirable, but one *could* measure a long distance by means of pacing out the distance and a short one by the number of palm breadths. All men haven't the same length of pace, nor have they the same breadth of palm. It is imperative therefore that, whatever the system of measurement, it should be standardized and should be such as to meet the practical requirements of those using it. Measurements made in different places at different times and by different people, should be intelligible one to the other.

2. STANDARDS OF MEASUREMENT.

There are two systems of measurement at present in common use :—

(1) **The English or F.P.S. System,** in which the fundamental units for length, mass and time respectively are the foot, pound and second.

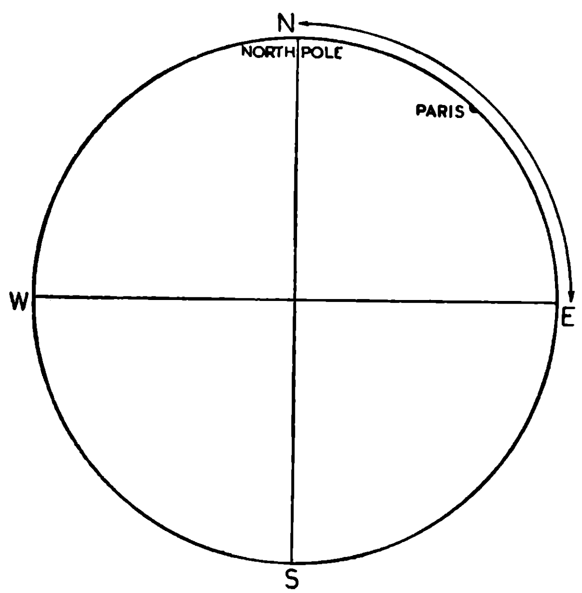
(2) **The Metric or C.G.S. System,** in which the corresponding fundamental units are the centimetre, gramme and second.

The Metric System is the one chiefly used in scientific work. It is a decimal system and hence, for calculation, has many advantages that the English System does not possess.

The English standard of length by law is **the Imperial yard,** which is defined as the distance, at a temperature of 60° Fahr., between the centre rulings on two gold plugs sunk in a bronze bar kept at the Board of Trade in London. The **foot** is one-third of the standard yard.

The Imperial standard **pound** is the mass of a certain piece of platinum also kept at the Board of Trade.

During the French Revolution, a special committee was appointed to determine the French standard of length and it recommended for adoption one ten-millionth of a quarter of the earth's circumference, measured along the arc of the meridian passing through Paris, between the Equator and the North Pole.



2. Standards of Measurement—*continued.*

Two transverse marks, separated by a distance equal to this length, were ruled upon a bar of platinum at the temperature of melting ice, and this distance was called **the metre.** The present **standard metre** is marked along a platinum-iridium bar kept in Paris. This length and the ten-millionth part of the earth's quadrant differ with slight error. The **centimetre** is one hundredth part of the standard metre.

There is in Paris, with the standard metre, a carefully preserved platinum-iridium cylinder whose mass denotes a standard **kilogramme.** It was intended originally that a kilogramme should be the mass of a litre (i.e., 1 cubic decimetre) of pure water, at a temperature of 4° C. The **gramme** is the one-thousandth part of the kilogramme.

The unit of time universally used in both the English and the Metric Systems is the **mean solar second,** which is $\frac{1}{86,400}$ $\left(\text{i.e., } \frac{1}{24 \times 60 \times 60}\right)$ of a mean solar day. It is based on the average time of one rotation of the earth on its axis relative to the sun as a point of reference. An average is taken because the length of day, as measured with reference to the sun, varies throughout the year. The mean solar day is divided into 24 hours, 24×60 minutes, $24 \times 60 \times 60$ seconds.

3. CONVERSION FROM ONE SYSTEM TO THE OTHER.

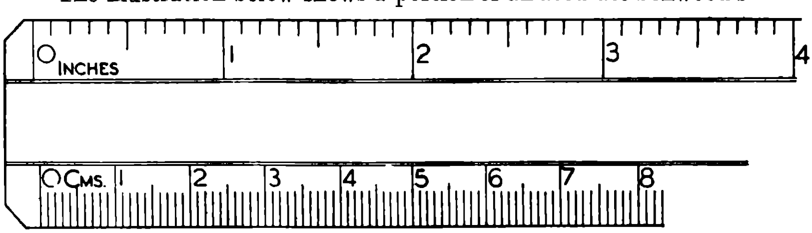
It is important to be able to convert the units of the one system into those of the other. For this reason, plenty of practice is given on such conversions in the Arithmetic section of the Mathematics course. You are strongly urged to commit to memory such relations as :—

1 inch	= 2·54 cm.	1 litre	= 1·76 pints.
1 metre	= 39·37 inches.	1 lb.	= 453·6 g.
1 kilometre	= 3,280 ft.	1 kilogramme	= 2·2 lb.
	= ·62 mile.		

4. ACCURACY, PARALLAX, THE VERNIER.

The accuracy of a measurement is limited by the accuracy of the instrument used. No instrument is perfectly accurate, although, of course, a high degree of precision is acquired by skilful workmanship.

The illustration below shows a portion of an accurate boxwood scale.



4. Accuracy, Parallax, the Vernier—*continued.*

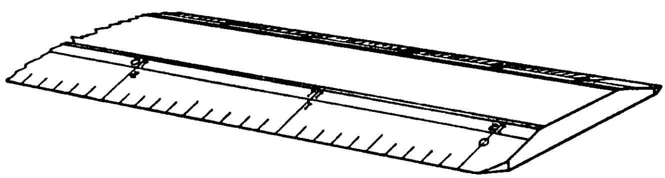
In making calculations we should always keep in mind that the degree of accuracy of the answer should be in keeping with the accuracy of the facts given in the problem.

Thus, it would be quite unacceptable to state the answer for a certain calculated length as 2·2645 inches just because the decimals appear more "accurate," whereas, from the facts given in the problem, it might be apparent that measurements of length can be read only to the nearest tenth of an inch. Consequently, in such a case, the answer should be given as 2·3 inches.

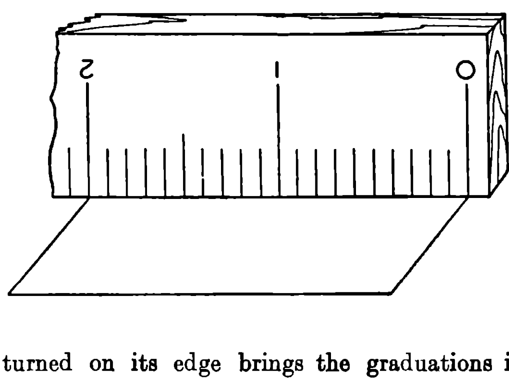
Precautions in using a scale to measure length.

(1) It is better to start measurements from some division other than the end of the scale, especially if the scale is at all worn at the ends.

(2) Always place the scale so that the divisions are as close as possible to the object to be measured.



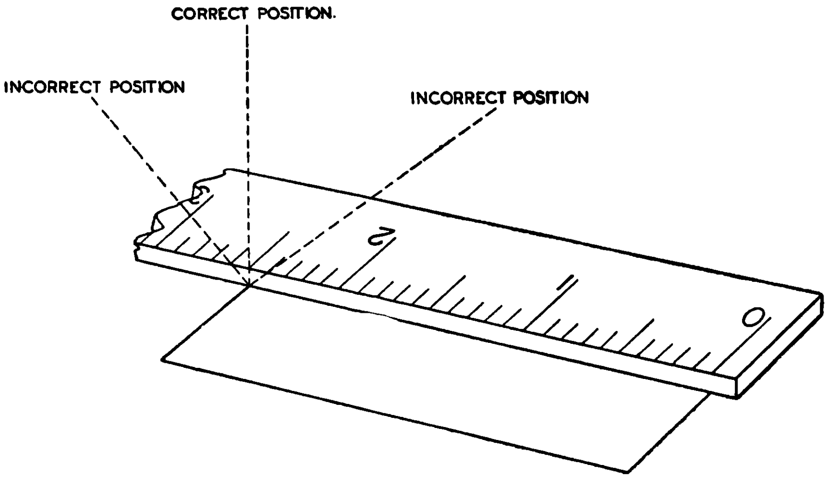
A bevelled scale brings the edge as close as possible to the body to be measured.



A scale turned on its edge brings the graduations into contact with the object to be measured.

4. Accuracy, Parallax, the Vernier—*continued.*

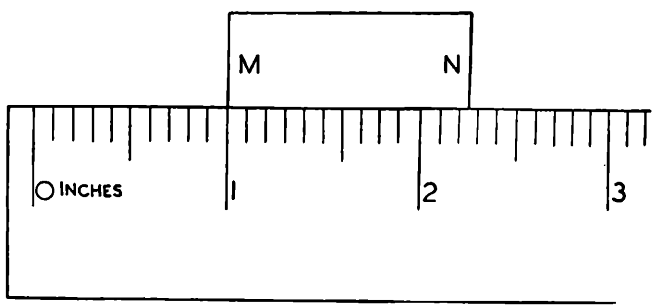
(3) When making scale measurements, be certain to look perpendicularly down on to the graduation to be read on the scale. Errors due to the incorrect position of the eye in reading scales are called **Errors of Parallax.**



The errors of parallax in the above illustration result in the true reading of 2·5 inches being given as 2·4 and 2·6 inches respectivly.

The Vernier.

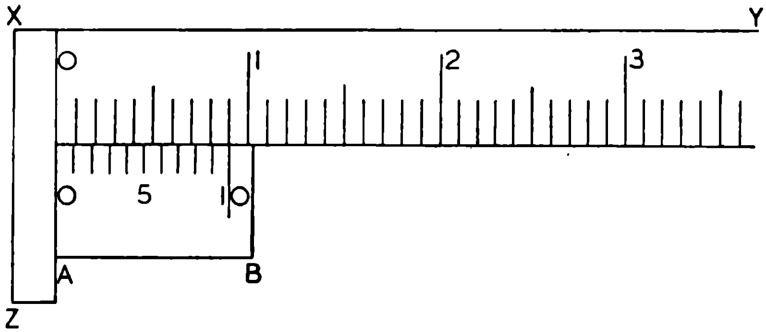
If we use a scale which is marked off in inches and tenths of inches for measurements of lengths, it is possible, with practice, to state the lengths to the nearest estimated hundredth of an inch. Thus, in the diagram, the length of MN may be estimated to be 1·25 inches.



The vernier provides us with a method of making such measurements accurately. To understand the action of the vernier, it is advisable to make one from stiff cardboard. Cut out a piece of rectangular card

4. Accuracy, Parallax, the Vernier—*continued.*

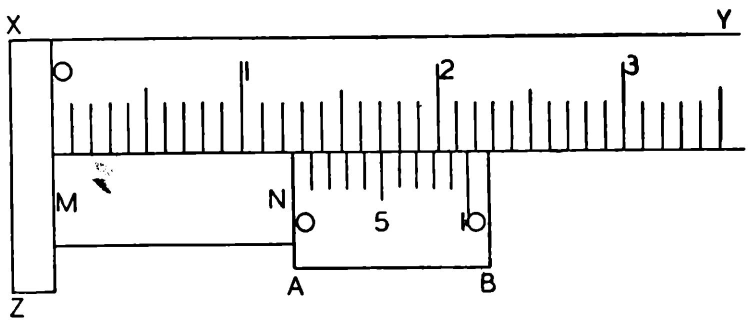
board XY as shown and mark off its length in inches and tenths of inches. (The portion XZ is at right angles to XY.) Take a second piece of rectangular cardboard AB and mark on it a length equal to



nine-tenths of an inch. Divide this length into ten equal parts. Then the total length of the ten vernier divisions is 0·1 inch less than the total length of the ten scale divisions, hence **each** vernier division falls short of **each** scale division by 0·01 inch.

To measure a length MN as shown:—

From the main scale, the reading is 1·2 inches, together with a portion of the next tenth division. Looking along the vernier, it



is seen that its sixth division coincides with a scale division. Six vernier divisions fall short of six scale divisions by 0·06 inch. Hence the portion of the tenth division on the scale concerned in the length of MN is 0·06 inch, i.e., the length of MN is 1·26 inches.

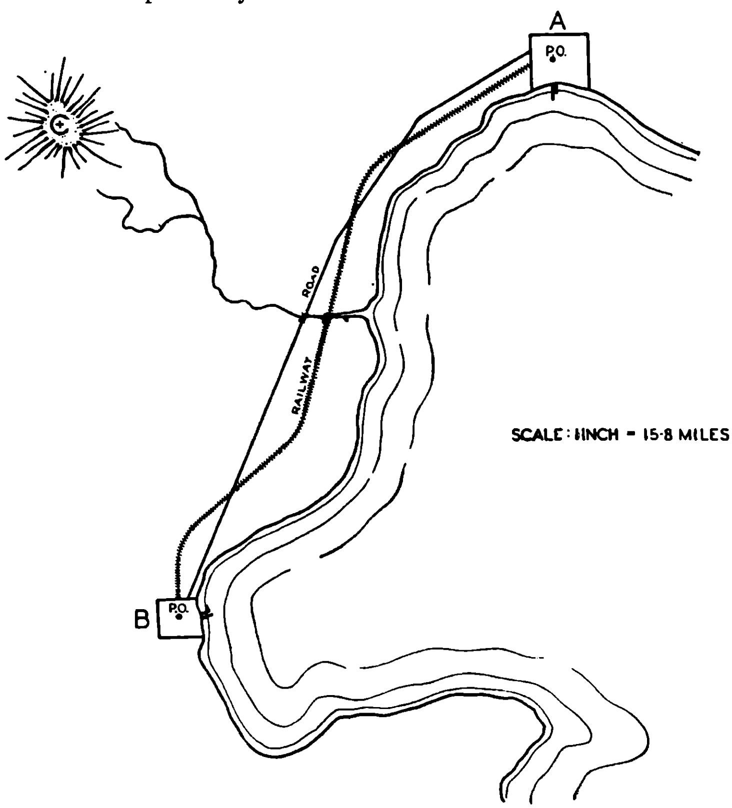
Note.—The vernier can be readily made to slide along XY either by forming runners pasted to it and passing over XY or by cutting two slits in the piece of cardboard on which the vernier is marked and allowing XY to slide through the slits.

EXERCISES.

1. Draw a straight line **about** 3 inches long. By means of a scale, measure its length to the nearest tenth of an inch, and estimate its length to the nearest hundredth of an inch. Move your eye both well to the right and to the left of the correct position for reading the scale and record the results thus obtained.

Measure the line also to the nearest millimetre. Calculate from your results how many cm correspond to 1 ″.

2. Draw a plan of your desk or table to a suitable scale.



3. The distance-scale for the accompanying map is, 1″ represents 15·8 miles. Measure, as accurately as your scale will allow, the distance in a straight line from the post office of town *A* to the post office of town *B*, these being represented by dots. If an aircraft were directly over the mountain *C*, which would be the closer town, *A* or *B*? State the difference in miles. How many kilometres is this?

For additional practice, similar readings can be made from other maps.

Exercises—*continued.*

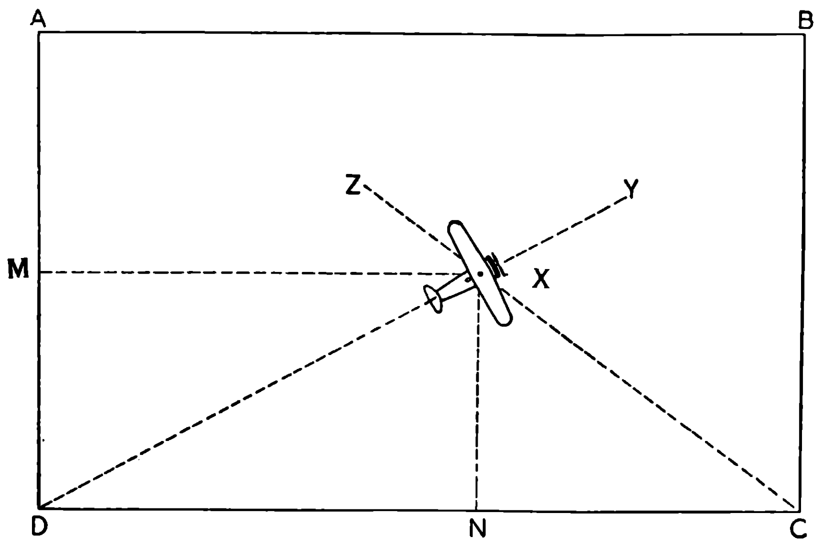
4. In addition to being able to measure distances accurately, it is also very desirable to be able to estimate them reasonably well. Judge the distances from your own position, to prominent objects such as church steeples and crossroads. If a map of your own locality is available or if there is some other way of so ascertaining, compare the correct distances with your estimates. Likewise, estimate heights whenever possible—of buildings, flag-poles, mountains, clouds, &c. Verify your efforts wherever possible.

It is also an exercise in judgment to estimate short intervals of time. A useful guide for estimating a number of seconds, up to a minute or more, is to count, "Tick-one, tick-two . . . . . . &c." With a little practice with a good timepiece, quite a reasonable degree of accuracy can be acquired.

MOTION.

1. POSITION.

Suppose, $ABCD$ in the diagram, represents a rectangular field and X an aircraft standing on it. The **position** of the aircraft can be fixed in various ways, some of which are mentioned here. If we take the lines AD and DC for reference, the two perpendicular distances MX, NX will completely fix the point X. Alternatively, the angle CDY together with the distance DX would locate it. Yet again, if we were told that the aircraft lay somewhere along DY, making an angle CDY with DC, and also along CZ, which made an angle DCZ with CD, we could say it was situated at the intersection of the two lines DY and CZ, i.e., at X.



The position of a body can be determined only in relation to other arbitrarily chosen points or lines of reference.

2. MOTION.

If the aircraft were to change its position **relative** to the chosen points or lines of reference, it would be said to be **in motion** relative to the points or lines of reference.

Sitting inside a railway carriage a person would be **at rest** with reference to the seat and the other carriage fittings, but he would be **in motion** with reference to telegraph poles outside, if the train were moving, i.e., if he were changing his position with regard to the poles.

3. DISPLACEMENT.

If the aircraft in the example above, in changing its position, moved along DY a distance of 100 feet from X to Y, it would be said to have undergone **a displacement** XY which represents the distance 100 feet.

To fully specify a displacement three things are required :—

1. The **magnitude** of the displacement, i.e., the length of the straight line joining its initial and final positions.
2. The **direction** in which the displacement takes place.
3. The **sign** of the displacement, i.e., whether it is positive or negative in the given direction, e.g., whether the displacement is from or towards a given point.

Much fuller reference will be made in regard to positive and negative signs in the Algebra course.

In the above example, the displacement of the aircraft is 100 feet (magnitude) in the direction of DY, which makes a given angle CDY with CD, a given line of reference, and is from D or towards Y (sign).

4. VELOCITY AND SPEED.

A displacement requires a certain time in which to take place. This brings us to the idea of **time rate of displacement** or **rate of displacement with respect to time.** If the aircraft we have been considering were pushed by ground staff from X to Y in 30 seconds, the rate of displacement would be 100 feet in 30 seconds or 200 feet per minute (or $3\frac{1}{3}$ ft. per sec.) in the direction DY. The men pushing the aircraft would be walking at the rate of approximately $2\frac{1}{4}$ m.p.h. in the direction specified. If the aircraft underwent the displacement under its own power, the rate of displacement could be much higher.

The **velocity** of a moving body is the rate of its displacement with respect to time.

To fully specify velocity we require :—

(1) The **magnitude** of the velocity, i.e., the distance covered by the body in a given time expressed as **a rate,** e.g., so many feet per second, feet per minute, miles per hour, &c.

(2) The **direction** in which the movement occurs, in relation to some given points or lines of reference.

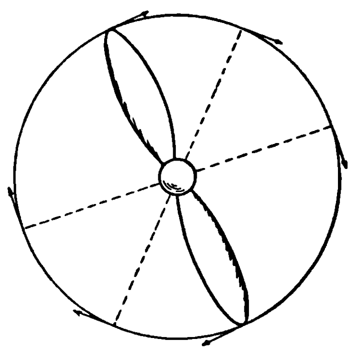
(3) The **sign** of the velocity, e.g., whether the body is going from or coming towards some given point.

4. Velocity and Speed—*continued.*

In everyday language we usually do not differentiate between the terms **velocity** and **speed,** but in Science we must make the distinction.

The **speed of a body** implies only the rate at which its position is changing, or its rate of movement, without any stipulation as to direction. Thus, we may speak of the **speed** of a body as 88 ft. per sec., but to give its **velocity** we should need to specify the direction as well—88 ft. per sec., say, due North.

It is possible for a body to be moving with constant speed but yet its velocity to be altering. For example, the speed of a stone, whirled in a circle at the end of a piece of string, may be kept constant, but its velocity will be continually changing because of the continual change of direction in describing its circular path.



As another example, we have the tip of a revolving propeller, the rate of rotation of which is kept constant. The tip is moving with constant speed but its direction is continually changing.

For calculating the magnitude of velocity or speed, the following rule is used :—

$$\text{Velocity (or Speed)} = \frac{\text{distance traversed}}{\text{time taken}}$$

From this we can also say—

$$\text{Distance} = \text{Velocity} \times \text{Time, or } \textbf{Time Taken} = \frac{\text{Distance}}{\text{Velocity}}$$

It is important to remember that the term **"Knot"** indicates a speed. A body moving at the rate of one knot covers **one nautical mile (6080 ft.) per hour.**

4. **Velocity and Speed**—*continued.*

A useful rule to remember is that 60 m.p.h. = 88 ft./sec.

or, $\frac{60}{88}$ m.p.h. $= \frac{15}{22}$ m.p.h. $= 1$ ft./sec.

i.e., To convert ft./sec. into m.p.h. multiply the number of ft./sec. by $\frac{15}{22}$.

To convert m.p.h. into ft./sec. multiply the number of m.p.h. by $\frac{22}{15}$.

EXERCISES.

1. Draw a straight line OX, 6 inches in length, across your page, and draw OY, also 6 inches long, at right angles to it up the page :—

(i) Locate the position of a point A whose perpendicular distance from OY is 3 inches and from OX, 4 inches.

(ii) Measure carefully the distance of A from O and the angle OA makes with OX. Is there any other point that is located by these two measurements ?

(iii) Supposing 1 inch in your diagram represents 12 miles on the ground. How far, in miles, would A be from O ? How far is this in kilometres ? If an aircraft covered an equivalent distance in 30 minutes, what would its average speed be, in miles per hour, feet per second, and kilometres per hour ? Specify also its average velocity, given that the aircraft was travelling from O to A.

Using the same diagram as for exercise 1, mark the point B along OX, 5 inches from O. At B draw an angle OBM equal to 60° so that the arm BM intersects in C a line drawn from O at an angle of 30° to OX.

Measure the length AC and, using the same scale as before, convert it to miles. If an aircraft travelled from A to C at a uniform velocity of 180 m.p.h., how long to the nearest minute would it take to cover the distance ?

3. If a man can run 100 yards in 10 secs., what is his average speed in m.p.h. ?

4. The following exercises are offered as suggestions, to be done according to the opportunities available. Where they are not available, you may be able to compile alternatives :—

(i) Ascertain your average rate of walking by noting the time it takes you to cover a known distance. Using this average rate, estimate the times taken to cover known distances and check on a watch.

(ii) If you could maintain approximately the same rate of walking, estimate how far you could cover in various time intervals.

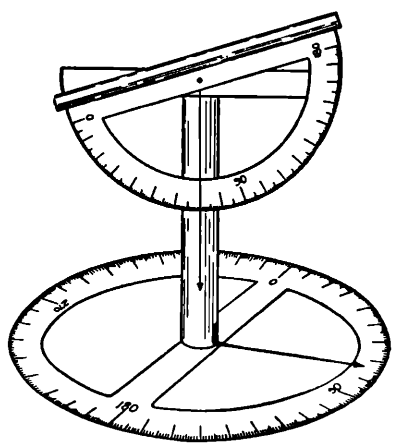
Exercises—*continued.*

(iii) Estimate the average speed of a train in which you may be travelling, by noting the time it takes to pass a number of mile-posts.

(iv) If an echo can be obtained, estimate the width of a valley, or the distance to a large building, by noting with a stop-watch the time it takes for a sufficiently loud sound to return. (The journey of the sound across the valley will take half the time observed. Assume that sound travels at 1,100 ft. per second and that the air is still.) If the distance to an object that produces an echo is known, estimates may be made of the speed of sound. It would also be useful practice to estimate the distance by eye before you perform your experiment.

5. A simple theodolite can easily be made, which can be used for additional exercises on this section of your course, as well as for other portions of the course. Even if you are unable to make one, it is interesting and instructive to understand its operation and use.

Fix a metal-tube parallel to the base of a blackboard protractor. To the centre of the base of the protractor, fix a cord, to the



lower end of which a weight is attached, to act as a plumb-line. Mount the protractor so that it can rotate freely on a support as shown. This support, whilst being at right-angles to a base-board divided into 360°, is itself capable of being rotated at the centre of the base. A pointer, attached to this support and parallel to the tube, can be made to move over the circular scale.

Exercises—*continued.*

Having made the theodolite, a few simple exercises are as follows:—

(i) Select a prominent object. From the base scale, read off the direction of the object from the true North–South line at a given locality. Walk, say, due East over a known distance and read off the direction of the object from the new position.

By making a drawing to scale, ascertain how far the object must be from each of the positions selected.

(ii) Select a tall object, such as a flag-staff. Walk a measured distance from its base. At this position read from the vertical protractor the angle through which the sighting tube has to be turned from the horizontal so that the top of the object is in sight.

By making a drawing to scale, ascertain the height of the object. Add the height the centre of the theodolite tube is above the ground.

Again you might first estimate the distance and height and compare your estimates with the results obtained with the theodolite.

Many other uses of the theodolite will suggest themselves during the course.

MOTION—(continued).

1. UNIFORM AND AVERAGE SPEED AND VELOCITY.

In the previous lecture we used the terms uniform and average speed (and velocity). The speed of a body is said to be **uniform** when it covers a certain distance in a certain interval of time, without altering its speed throughout that interval. If we are told, however, that a man walks 10 miles in $2\frac{1}{2}$ hours we can only really say that he averages 4 miles per hour. During the journey his speed may have been greater than 4 m.p.h. for some of the way (say 5 m.p.h.), or it may have been less, e.g., 3 m.p.h., but we can say his **average speed** is 4 m.p.h.

The terms **uniform velocity** and **average velocity** are, of course, also employed and have meanings analogous to those for speed.

2. UNIFORM ACCELERATION.

Suppose a tram car, starting from rest, acquired a velocity of 15 m.p.h. in 10 seconds in a certain direction, e.g., down a long straight street. Assuming this velocity was acquired uniformly, we can say that the velocity increased at the **rate** of 1·5 m.p.h. every second. At the end of one second the velocity was 1·5 m.p.h., at the end of the second second, 3 m.p.h., at the end of the third, 4·5 m.p.h., and so on, all in the given direction.

As another case, suppose a train had a velocity of 22 feet per second, at a certain instant, and after 12 seconds its velocity was gradually and uniformly increased to 82 feet per second. The velocity of the train was thus increased 60 feet per second in 12 seconds. In each second the velocity added was 5 feet per second, or in other words, the **time rate** of increase in velocity was 5 feet per second per second. At the end of one second from when observations began, the velocity was 27 feet per second, at the end of 2 seconds, 32 feet per second, and so on. **The rate of change of velocity with respect to time is termed acceleration.** The acceleration of the tram car, in our first example, was 1·5 m.p.h. per second (or, 2·2 feet per second per second, sometimes written 2·2 feet/sec.^{2}), whilst that of the train in the second example was 5 feet per second per second.

An acceleration may be **positive,** when the velocity is increasing, or **negative,** when it is decreasing. A negative acceleration is also called a **retardation.** If a ball is thrown into the air, it will be noticed that it will gradually slow down, reach its highest point and then proceed to descend with gradually increasing velocity. It is said to undergo a negative acceleration (or a retardation) on its upward journey and a positive acceleration on its downward journey. A further study will be made of these effects in a later lecture.

2. Uniform Acceleration—*continued.*

To fully understand the meaning of acceleration, we require :—

(1) The **magnitude** of the acceleration, i.e., the time rate at which the velocity is changing.

(2) The **direction** in which the acceleration is taking place.

(3) The **sign** of the acceleration, i.e., positive or negative.

From the definition of acceleration given above, it is apparent that an acceleration may occur in three ways.

(1) The **direction** of the velocity **remains constant** but its **magnitude changes.**

(2) The **magnitude** of the velocity **remains constant** but its **direction changes.**

(3) Both the **magnitude and the direction** of the velocity **change simultaneously.**

3. EQUATIONS OF MOTION.

For conciseness and convenience, it is sometimes an advantage to express, in symbolical form, the results we have learnt. In relation to problems on motion, the symbols very commonly employed are:—

v for **final velocity** acquired by a body after a certain time interval, for example, the tram car mentioned above, acquired a velocity v, 15 m.p.h. after 10 secs.

u for **initial velocity** possessed by a body at the beginning of an observation, for example, the train mentioned above started with a velocity u of 22 feet per sec.

a for acceleration.

t for time.

s for distance.

v is also used for uniform velocity.

N.B.—The choice of units for the terms depends upon the problem, but it is very important to be consistent in their use in any one problem.

In lecture 2 it was seen that, if a body is moving with uniform velocity, the distance travelled may be expressed as

$$\text{Distance} = \text{Velocity} \times \text{Time}$$

Using the symbols given above, this equation may be expressed as

$$s = vt$$

Again, we have just learnt that the time rate of change in velocity, or

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken for that change}}$$

3. Equations of Motion— *continued.*

If u = the initial velocity and v = final velocity, then the change in velocity $= v - u$

$$\text{and so} \quad a = \frac{v - u}{t}$$

If u is greater than v, we obtain a negative result for the equation. i.e., a is a retardation.

If the body starts from rest, $u = 0$, and so

$$a = \frac{v}{t}$$

As another example :—

If a body has an initial velocity, u, and this is uniformly increased to a final velocity v, then the average velocity, for the interval of time concerned, is $\frac{u + v}{2}$ in appropriate units.

The distance s covered, in time t, at this average velocity would be :—

$$s = \frac{u + v}{2} \times t$$

Thus, suppose a body, starting with a velocity of 5ft./sec., acquired a velocity of 10 ft./sec., after 3 seconds.

The average velocity is $\frac{5 + 10}{2}$ ft. per second, i.e., $7\frac{1}{2}$ ft./sec. and the distance travelled is $(7\frac{1}{2} \times 3)$ ft., i.e., $22\frac{1}{2}$ feet.

If the body starts from rest, $u = 0$, and the equation for the distance s covered becomes :—

$$s = \frac{v}{2} \times t$$

Other equations of motion can also be deduced. A section is set aside in Lecture 9 of Air Crew Mathematics, 1B/2a/1 Algebra, which deals with these equations after the necessary mathematical processes have been covered.

A list of the formulae relating to motion already established, is set out hereunder :—

Bodies having a uniform velocity.

$$s = vt$$

Bodies starting from rest.	*Bodies having an initial velocity.*
$a = \frac{v}{t}$	$a = \frac{v - u}{t}$
$s = \frac{v}{2}t$	$s = \frac{u + v}{2}t$

where v, u, s, a, and t, have the meanings indicated above.

EXERCISES.

1. An express train left a certain station at 10 a.m. (1000 hours on the Continental or 24 hour system of expressing time), and arrived at its destination 400 miles away at 6.30 p.m. (or 1830 hours). What was its average speed in m.p.h. ? During one stage of its journey it covered a distance of 6 miles in 5 minutes at a uniform speed. What was this speed in m.p.h. and in ft./sec. ?

2. A motor car increases its velocity uniformly in a given direction from 16 m.p.h. to 28 m.p.h. in 2 minutes. What is its acceleration in miles per hour per second, miles per hour per hour, and in ft./sec.^{2} ? How far did the car travel in the time specified ? (Answer to the nearest yard.)

3. A train was travelling at 30 m.p.h. and its velocity was uniformly reduced to 15 m.p.h. in 22 secs. What was the retardation in ft./sec.^{2} ?

4. A body is moving with an acceleration of 32 ft./sec.^{2}. Assuming it started from rest, what would its velocity be after 1 second, 2 seconds, 5 seconds ? What would these values be if it had an initial velocity of 32 ft. per second ?

5. During a scientific experiment, the flash was observed from a gun fired at night. It was recorded that the sound of the gun explosion was heard 66 seconds after the flash was seen. The gun was known to be on a hill 14 miles away. Calculate the speed of sound as determined by these figures (in ft./sec.).

6. Comment on the relative magnitudes of the following accelerations :—

   (i) One foot-per-second per ten seconds.
   (ii) One foot-per-ten-seconds per second.
   (iii) One tenth of a foot-per-second per second.

7. You are strongly advised to get into the habit of looking for practical applications of principles you learn. There are many if only you will do so. The work you are being taught involves either knowledge you may actually employ in the Air Force or may require to understand other instruction.

   e.g., From a railway time table you can work out average speeds of various trains. You can calculate the acceleration of a motor car in which you are travelling. You can estimate the distance of a thunderstorm on the principle of question 5. (Estimate the time if you haven't a watch handy.)

INERTIA, MASS, FORCE.

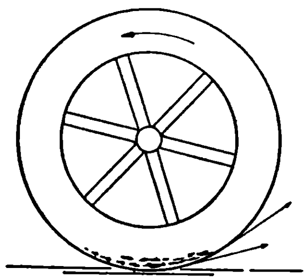
1. INERTIA.

Consider first the following everyday occurrences :—

(*a*) A man standing in a tram car has a tendency to fall towards the front of the tram when it is suddenly braked. If it were to start again suddenly he would tend to topple towards the rear.

(*b*) A horse endeavouring to set a heavy load in motion, has to put forth a big effort at the outset, but once he has set the load in motion he can keep it going with reduced effort. Indeed, to stop or slow up the load he would have to push **back** hard.

(*c*) If the mud on a rotating bicycle wheel is no longer able to adhere to it, it flies off at the point of its release in a straight line which is a tangent to the wheel.



Mud leaving the wheel.

(*d*) An athlete, swinging a hammer around his head before throwing it, finds he has to pull inwards to prevent the hammer from flying off at a tangent. When he does release the pull inwards, the hammer moves off in a straight line, a tangent to the circle of its former motion.

In the first example above, the man tends to keep moving in a straight line with the velocity he possessed prior to the tram stopping suddenly. If he is in a state of rest when the tram starts, he tends to remain at rest until compelled to change that state.

The load referred to in the second example tends to remain at rest, but once it is set in motion it tends to keep moving in a straight line.

At any instant, the mud on a rotating wheel is moving in a straight line which is a tangent to the wheel. Immediately the mud is released from the wheel and so is not compelled by adhesion to follow a circular path, it moves along the tangent at its point of release. A similar explanation may be found for the example of the hammer.

1. **Inertia**—*continued.*

The above examples are illustrations of the effects produced by the **Inertia** of a body, which may be defined as follows:—

> That property of a body by virtue of which it tends to remain in its state of rest, or of uniform motion in a straight line, is termed **Inertia.**

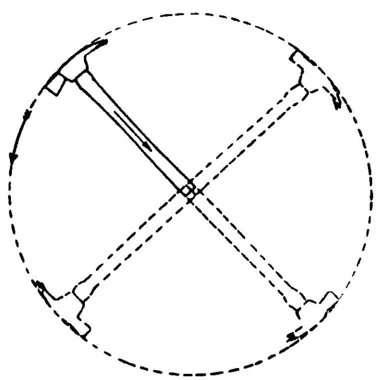
2. **MASS.**

The mass of a body is commonly defined as the quantity of matter it contains and is that property which causes a body to possess inertia. The mass of a body may be regarded as an invariable quantity. The units of mass have been defined in Lecture 1.

3. **FORCE.**

Force is generally defined as that which changes, or tends to change, the state of rest of a body or its uniform motion in a straight line.

For example, the pull that had to be exerted by the horse (Example (*b*) above), to set the load in motion, and the push required to stop or slow up the moving load, are examples of forces. Again, the inward pull exerted by the athlete (Example (*d*) above) on the hammer, to compel it to follow its circular path, is another example. This latter force is given a particular name. It is called a **centripetal force.** Further reference will be made to such forces later.



Centripetal force inwards towards centre.

If a man pushes on a body too heavy for him to set in motion, he is still said, nevertheless, to be exerting a force, and it is also apparent that the body is pushing on the man with an equal and opposite force.

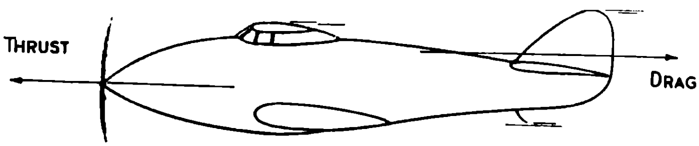
3. **Force**—*continued.*

If a vehicle is moving with uniform motion in a straight line, a force has to be exerted to increase or decrease the magnitude of its velocity. If the velocity is increased, it is said to undergo an acceleration; if it is reduced, a retardation. A force is necessary also to change the direction of the motion.

How are these forces exerted, in increasing and decreasing the speed of a motor car and in changing its direction of motion ?

Again, if a train is proceeding at a uniform velocity, forces are acting on the train tending to reduce this speed. Among these forces are those due to air resistance and friction. What then maintains the constant speed ? The locomotive has to exert a pull sufficient to overcome the forces resisting the train's motion. What would happen if the locomotive exerted a force greater than that just required to overcome the total resistance ?

As we shall see later, the thrust of a propeller must be equal and opposite to the drag on an aircraft to maintain uniform horizontal flight. It is enough to say, at this stage, that the drag on an aircraft is due to the resistance offered by the exposed parts of the aircraft to the airflow past those parts.



From these examples, it is apparent that when a force acts on a body, it tends to produce acceleration and, conversely, if a body is accelerated, then force must be acting on it.

4. LAWS OF MOTION.

Sir Isaac Newton, (born in England, 1642), investigated the relations of these fundamental ideas of Inertia, Mass, and Force with each other and also with length and time. He enunciated certain statements concerning these relations which are known as the :—

LAWS OF MOTION.

We are in the position to give the formal statement of the first of these laws at this stage.

Newton's First Law of Motion.

Every body continues in its state of rest or of uniform motion in a straight line unless compelled by external forces to change that state.

The remaining two laws of motion will be taken in their context in Lecture 6.

EXERCISES.

1. If a small coin is laid on a visiting card placed over a wine glass and the card is sharply flicked away by a horizontal blow from a finger nail, the coin drops into the glass. Explain fully.

2. The rider of a bicycle may be thrown over the handle bars if his machine is stopped too suddenly, say, by striking the kerb. Why?

3. A thread attached to a heavy article, such as an electric iron, may snap if pulled sharply, but may be able to set the load in motion if pulled slowly. Why?

4. When alighting from a moving train a guard leans sharply back. Why? Explain also why notices are frequently displayed in tram cars advising you to alight facing the direction in which the tram is going.

5. Why does the dust leave a carpet when it is hung over a line and beaten?

GRAVITY, WEIGHT.

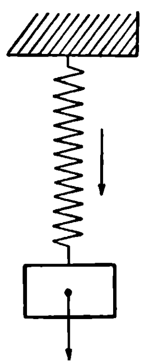
1. GRAVITY.

If an object slips over the edge of a table, it falls vertically downwards to the floor.

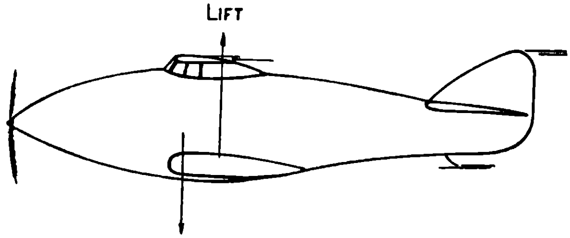
A builder to test perpendicularity uses a plumb-line, which is made by suspending a weight to the lower end of a cord tied to a suitable support. The weight pulls the cord taut into a vertical position.

A ball thrown vertically upwards moves with gradually diminishing velocity until it momentarily comes to rest, whereupon it starts to move vertically downwards with gradually increasing velocity.

These observations seem to indicate that there is a force acting between the earth and bodies near to it and, from the examples given above, it appears that it is a pull acting on the bodies vertically downwards towards the centre of the earth. This force acting between the earth and the bodies, is a force of attraction and is called the **FORCE OF GRAVITY.** This force is not the same for all bodies; for example, note the differences in extension produced by different bodies on a spring balance.



Extension produced in a spring by a weight.



The lift exerted on an aircraft in horizontal flight, due to its motion through the atmosphere, balances the weight of the aircraft

2. LAW OF UNIVERSAL GRAVITATION.

Sir Isaac Newton also investigated gravitational effects and stated the Law of Universal Gravitation :—

> Any two masses attract one another with a force which is directly proportional to the product of the masses and inversely proportional to the square of their distance apart.

If you do not yet understand fully the mathematical significance of the terms denoting proportionality, it might be said that the Law states that gravitation exists between any two masses in the universe, the larger the masses (or more accurately, the product of the masses), the greater the force of attraction, and the less the distance between them, the greater the force.

If we double the distance between them, the force is only one-quarter as great, and so on.

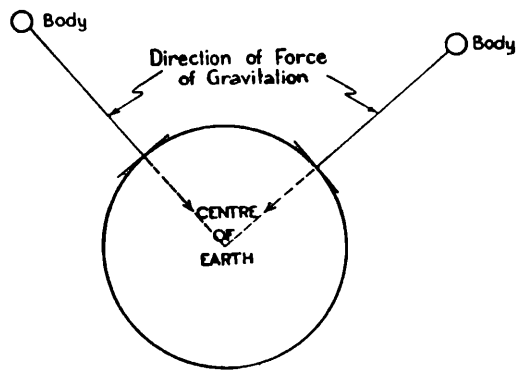
This may be expressed in symbols thus :—

$F = K \frac{M_1 M_2}{D^2}$ where F = force of attraction.

M_1 and M_2 = Masses involved.
D = Distance between them.
K = Gravitational constant.

3. WEIGHT.

The gravitational force exerted by the earth on a body is termed its **weight.** This force acts vertically downwards towards the centre of the earth. The terms weight and mass must not be confused. Weight as we have just said is a **force.** The **mass** of a body, as stated in Lecture 3, is the quantity of matter in it. The mass of a body does not vary from locality to locality on the earth's surface, but the weight may. The earth is not a perfect sphere. The poles are slightly closer to the centre of the earth than points on the equator. Consequently the weight of a body is less at the equator than at the poles, the difference amounting approximately to 0·5%. The summits of mountains being further from the centre of the earth than the valleys below, there will be a similar variation in the weight of a body according to whether it is weighed at the mountain top or in the valley. The mass of any given body is determined by balancing it against standard masses.



2. **Law of Universal Gravitation**—*continued.*

The same names are used for the units of mass and weight. Thus we speak of 1 lb. (or 1 gm) mass and 1 lb. (or 1 gm) weight.

4. **ACCELERATION DUE TO GRAVITY.**

In a freely falling body, it is the force of gravity that produces the acceleration or in the case of a body projected upwards, the retardation.

The value of the acceleration due to gravity is approximately 32·2 feet per second per second or 981 cm per second per second. These values vary according to latitude on the earth's surface and are greatest at the poles. The acceleration due to gravity is usually denoted by the symbol g.

All bodies fall at the same rate in a vacuum, irrespective of their weights. In air they may fall at different rates, if the areas they present to air resistance differ. Thus a coin and a feather fall at the same rate in a vacuum, but the coin falls the faster in air.

Before the time of the great Italian philosopher, Galileo (1564–1642), it was generally believed that heavy objects fell faster than lighter ones. To convince the people otherwise, Galileo dropped balls of different sizes and materials from the top of the leaning tower of Pisa. The results of the experiment confirmed his views, but there were still some who distrusted the evidence of their own senses, and continued to accept the theory of the old Greek philosopher, Aristotle, that the heavier objects fell faster.

The equations of motion apply to motion under gravity. Instead of "a," we can read "g," which is a constant for a given locality. It must be kept in mind that "g" has a negative value for a body projected upwards, the velocity of which at its highest point is zero.

Further practice will be given on these formulae in Lecture 9, 1B/2a/1 Algebra.

It should be noted that the velocity of a body falling through air from a great height does not continue to increase indefinitely. The air resistance increases as the velocity of the body increases, and as a result, there will be a stage at which the air resistance will equal the weight of the body, whereupon the body will fall with uniform velocity, which is called the **Terminal Velocity.**

The terminal velocity for a person falling freely can be taken as approximately 120 m.p.h. or 176 ft./sec., whilst that of an average bomb let fall from 10,000 feet is about 780 ft./sec. A parachutist lands with a velocity approximately equal to that which he would acquire, if he jumped from a wall about 15 feet high. Can you calculate what this velocity is ?

EXERCISES.

In the following problems, the air resistance to a falling body is neglected.

1. Given that the radius of the earth is 4,000 miles and that a body weighs one pound at the surface of the earth, to what height would it have to be conveyed for the weight to be only 4 ozs. ?

2. If two sheets of paper of the same size are taken and one is rolled into a tight ball, what is the result of letting both the ball and the sheet fall from the same height ? Explain.

3. What force prevents the moon from flying off at a tangent to its orbit ? Discuss.

4. Where would the force of gravity be greater—at the surface of the moon or at the surface of the earth ? Explain.

Between the earth and the moon there is a point where the weight of a body would be zero. Why ? Would this point be closer to the earth or the moon ?

5. An object is projected vertically upwards with an initial speed of 128 feet per second. What would be its speed after 1 second, and after 2 seconds ? How long will it be before it reaches its highest point ? What will be its velocity on arriving back at the point of projection ?

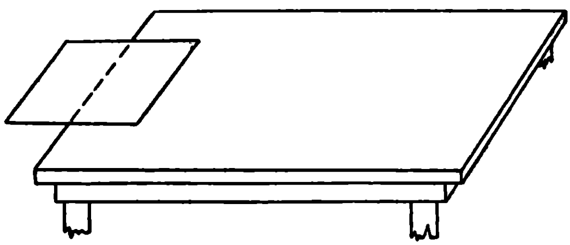
6. A body is let fall from an aeroplane. What would be its velocity at the beginning of the fourth second and at the end of the fourth second. What would be the average velocity during the fourth second ? How far would the body fall in this second ?

7. Can you suggest another additional reason, other than the one given in the lecture, why a body weighs less at the equator than at the poles ?

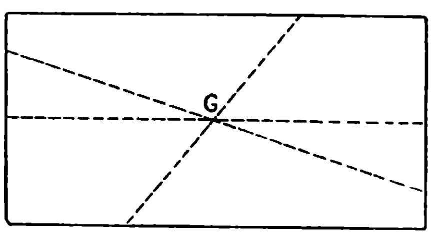
CENTRE OF GRAVITY—UNITS OF FORCE.

1. CENTRE OF GRAVITY.

Suppose a stiff piece of cardboard is laid on a table, possessing a well-defined straight edge, and is gradually slid over the edge, until it is on the verge of falling over. (See sketch.) Suppose too the cardboard is held firmly in this position and a straight line is drawn on its under-surface, to mark the edge of the table. If this process be repeated for any two other positions of the cardboard, it should be found that the three lines thus drawn intersect in a



point G. (See sketch.) If the experiment is performed accurately, it should be possible to balance the piece of cardboard at this point on say the tip of a pencil. Further, if the cardboard be pierced at

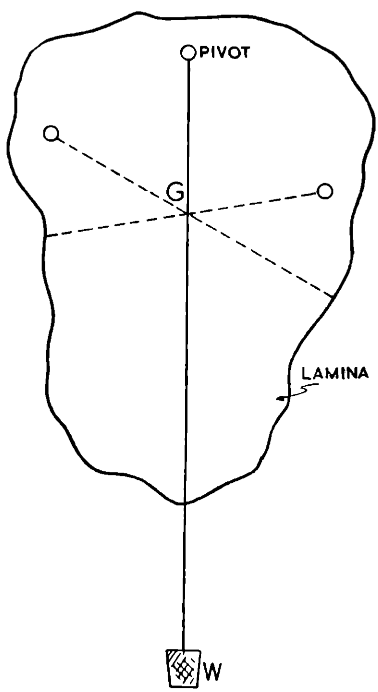


this point and mounted on a pivot, so that it would be free to move in a vertical plane, it should be found to remain in whatever position it is placed.

The point G that can be determined in this way is called **the Centre of Gravity** of the piece of cardboard. It can easily be verified that the cardboard falls off the table so soon as the centre of gravity projects over the edge.

1. Centre of Gravity—*continued.*

If the piece of cardboard is mounted on a pivot at points other than the centre of gravity, it will be found that it will rotate on the pivot until the centre of gravity is vertically below the point

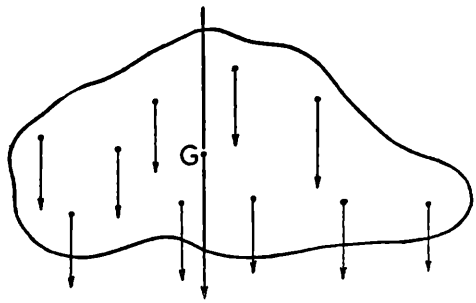


of support. The fact that the centre of gravity will come to rest vertically beneath the pivot provides us with an alternative method for finding the centre of gravity of a lamina. Suppose three holes are pierced near the boundary of a lamina and it is suspended, so as to move in a vertical plane, on a pivot through each of the holes in turn.

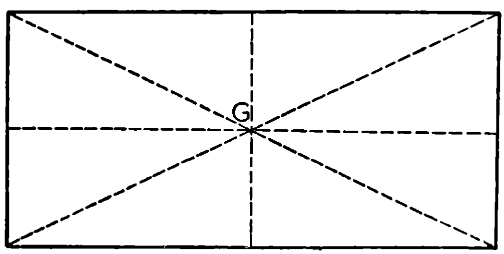
If a plumb-line is hung from the pivot, in front of the lamina, for each suspension and the direction of the thread of the plumb-line is carefully marked on the lamina in each case, it should be found that the three lines so obtained intersect at a point, which is the centre of gravity of the lamina. Two lines would be sufficient but the third is taken as a check.

1. **Centre of Gravity**—*continued.*

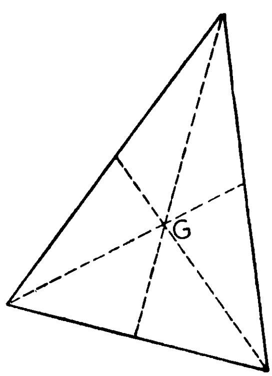
As we have said, the weight of a body is a force. Each particle of a body is acted upon by gravity and has a weight of its own. The sum (or resultant of these individual forces) is the weight W of the body. This force which is the weight of the body, acts through the centre of gravity G of the body.



3. **EXAMPLES OF THE POSITION OF THE CENTRE OF GRAVITY.**

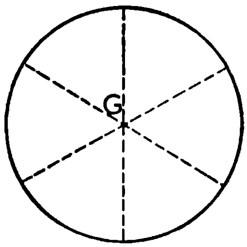


Rectangle

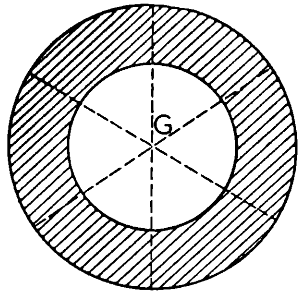


Triangle

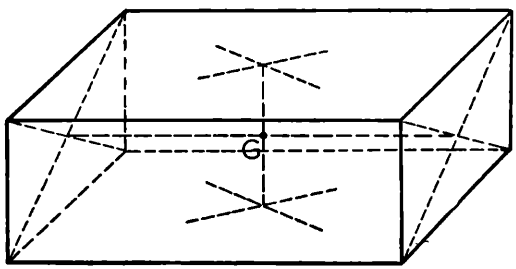
3. Examples of the Position of the Centre of Gravity—*continued.*



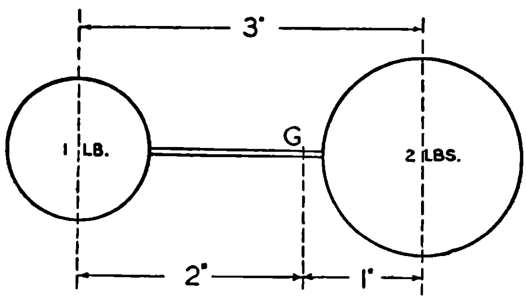
Circle



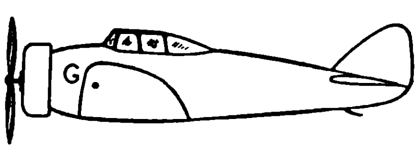
Circular Ring



Rectangular Solid



Two connected spheres. (Ignore weight of connecting rod.)



Aeroplane

4. THE UNITS OF FORCE.

The early experimenters on what they termed "the effect" of a moving body used the term **momentum**, which was defined as the product of its mass and its velocity.

As was shown in Lecture 3,

$$\text{the time rate of change in velocity} = \frac{v - u}{t}$$

Hence, the time rate of change in the momentum of a moving body may be expressed as $m\dfrac{v-u}{t}$, where m is the mass of the body.

Newton's Second Law of Motion states :—

> The time rate of change in momentum of a body is directly proportional to the force causing it and takes place in the direction of that force.

As already implied, a force can be measured by the acceleration produced in the body on which it is acting.

> It should be recognized that the time rate of change in velocity of a body is only another way of defining its acceleration. Thus, if appropriate units be chosen, the second law of motion may also be expressed in the form of the equation :—
>
> $$\text{Force} = \text{mass} \times \text{acceleration}.$$
> $$F = ma.$$

Unit force is the force that imparts unit acceleration to unit mass. The British absolute unit of force is the **poundal.** It is the force which imparts to 1 lb. mass an acceleration of 1 ft./sec.^{2}.

Now, if a body of 1 lb. mass were allowed to fall freely, it would have imparted to it an acceleration of 32·2 ft./sec.^{2} by the force acting on it, which as we now know, is its weight. This force is, therefore, 32·2 times as great as the absolute unit.

The weight of 1 lb. = 32·2 poundals (approx.),

i.e., 1 poundal = ½ oz. weight (approx.).

In the Metric System, the unit of force is the **Dyne.** It is the force which imparts to a mass of 1 gramme an acceleration of 1 cm per second per second. The force of 1 gramme weight is equal to 981 dynes approximately, i.e., the dyne is nearly equal to the weight of 1 milligramme.

SUMMARIZING.

BRITISH SYSTEM : The units of force are the **poundal** and the **weight of 1 lb.**

The weight of 1 lb. = 32·2 poundals approximately.

Summarizing—*continued.*

METRIC SYSTEM : The units of force are the **dyne** and the **weight of 1 g.**

The weight of 1 g = **981 dynes** (approx.).

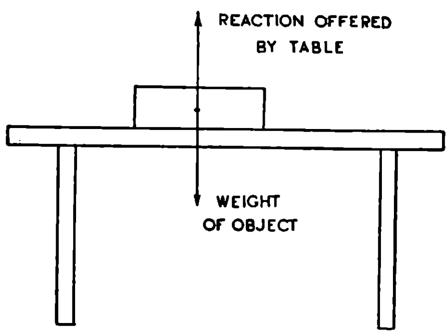
Note.— The pound weight and the gramme weight are the commonly used units of force in engineering work.

5. Newton's Third Law of Motion is as follows :—

To every force (or action) there is always an equal and opposite force (or reaction).

There are many examples of this law readily available.

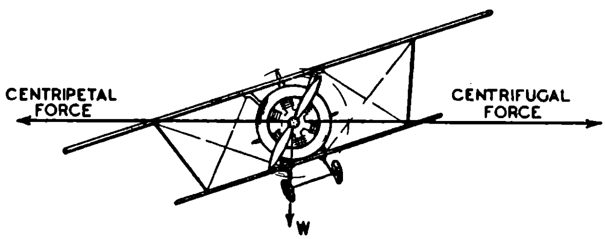
An object lying on a table pushes downward on the table, because of its weight. The table pushes upwards on the object with an equal and opposite force. If it is unable to do this, it will collapse under the weight of the object. A man in lifting a load exerts a force upward on the load. There is an equal and opposite force exerted on the ground by his feet. A bullet, in being propelled from a gun, has a force forwards exerted on it by the explosion. The gun is pushed backwards with an equal and opposite force.



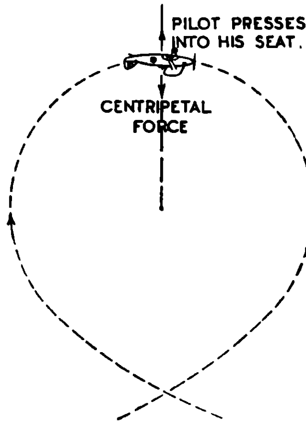
The weight of a body, as we have said, is the force exerted on it by the earth. The body exerts an equal and opposite force on the earth. This force is so comparatively small and the earth is so large that there is no perceptible movement of the earth to the body.

5. **Newton's Third Law of Motion**—*continued.*

To the centripetal force referred to in Lecture 4, there is also an equal and opposite force **outwards** along the handle of the hammer. This force is called a **centrifugal force.** There has to be a centripetal force exerted on an aircraft in turning. The aircraft exerts an equal and opposite force on the air.



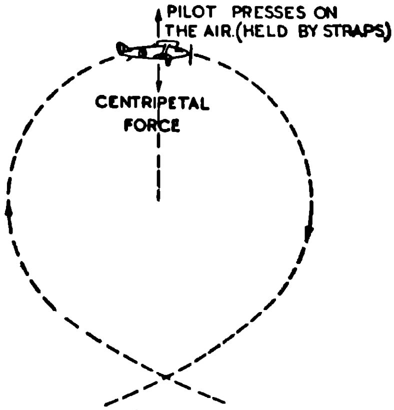
A pilot executing an ordinary loop is acted upon by the seat of the aircraft and equally he re-acts on the seat. **He** can feel a force and **the seat** also feels a force. See the diagram accompanying.



THE 'INSIDE' LOOP.

5. **Newton's Third Law of Motion**—*continued.*

The loop referred to is called an "inside" loop. There is a second form called the "outside" loop. From the diagrams, which would you consider the more dangerous form of loop?



EXERCISES.

1. Cut a stiff piece of cardboard into a triangular shape. Using both sides for ruling the necessary lines, find the centre of gravity of the triangle by the two methods given above. By piercing the cardboard with a pin, test if the two points so obtained are directly opposite each other. If they are not, test which method proved the most accurate by endeavouring to balance the triangle on the tip of a pencil.

2. Where, approximately, is the centre of gravity of a tea-cup?

3. Why is it hard to balance, say, a cork on the tip of a pencil, and yet it is comparatively easy when two forks slanting downward are stuck into opposite sides of it?

4. Convert :—

(i) A force of 100 gm wgt. to dynes.

(ii) A force of 64 poundals to lb. wt.

(iii) A force of 1 lb. wgt. to dynes. Hence, how many dynes are equivalent to 1 poundal?

5. A body whose mass is 100 lb. is observed to have an acceleration of 5 ft./sec.^{2}. What is the force acting on the body? Give the force both in lb. wgt. and poundals.

6. When discharging water on to a fire, why does a fireman have to hold the hose very firmly?

7. What causes a lawn sprinkler to rotate?

Exercises—*continued.*

8. A simple toy, if you have the material for making it, illustrates the principle of action and re-action. Take a strong toy engine, the spring of which is fully wound, and place it on a circular track mounted on a light base that is capable of rotating.

Observe what happens :—

(i) When the engine is placed on the track;
(ii) When the track is held fast;
(iii) When the engine is held fast.

Explain fully. Even if you haven't the apparatus, endeavour to reason the effects.

9. Why is the centre of gravity usually nearer to the front of an aircraft ?

6. REVISION SUMMARY.

1. 1 inch $= 2{\cdot}54$ cm.
1 metre $= 39{\cdot}37$ inches.
1 kilometre $= 3{,}280$ feet.
$= {\cdot}62$ mile.
$= \frac{5}{8}$ mile approximately.
1 litre $= 1{\cdot}76$ pints.
1 lb. $= 453{\cdot}6$ g.
1 kilogram $= 2{\cdot}2$ lb.

2. $$\text{Velocity (or speed)} = \frac{\text{distance traversed}}{\text{time taken}}.$$

Distance = velocity × time taken.

$$\text{Time taken} = \frac{\text{distance traversed}}{\text{velocity}}.$$

3. 1 knot = one nautical mile per hour.
1 nautical mile = 6080 ft.

4. To convert ft./sec. into m.p.h., multiply the number of ft./sec. by $\frac{15}{22}$.

5. To convert m.p.h. into ft./sec., multiply the number of m.p.h. by $\frac{22}{15}$.

6. $$\text{Acceleration} = \frac{\text{change of velocity}}{\text{time taken for that change}}.$$

in symbols, $a = \dfrac{v - u}{t}$.

7. Formulae relating to motion :—

$$a = \frac{v}{t} \qquad a = \frac{v - u}{t}$$

$$s = \frac{vt}{2} \qquad s = \frac{u + v}{2}t$$

$$s = vt \text{ (for uniform velocity).}$$

6. **Revision Summary**—*continued.*

8. **Inertia.**

That property of a body by virtue of which it tends to remain in its state of rest, or of uniform motion in a straight line, is termed inertia.

9. **Mass.**

The mass of a body is the quantity of matter it contains and is that property which causes a body to possess inertia.

10. **Force** is that which changes, or tends to change, the state of rest of a body or its uniform motion in a straight line.

11. **NEWTON'S LAWS OF MOTION.**

First Law :

Every body continues in its state of rest, or of uniform motion in a straight line, unless compelled by external forces to change that state.

Second Law :

The time rate of change in momentum of a body is directly proportional to the force causing it and takes place in the direction of that force.

In symbols, the law may be also expressed,

$$F = ma.$$

Third Law :

To every force (or action) there is always an equal and opposite force (or reaction).

12. **LAW OF UNIVERSAL GRAVITATION.**

Any two masses attract one another with a force that is directly proportional to the product of the masses and inversely proportional to the square of their distance apart.

In symbols,

$$F = K \frac{M_1 M_2.}{D^2}$$

13. **ACCELERATION DUE TO GRAVITY.**

$g = 32{\cdot}2$ ft./sec.^{2} or 981 cm/sec.^{2}, approximately.

14. **UNITS OF FORCE.**

BRITISH SYSTEM :

The poundal and weight of 1 lb.

Weight of 1 lb. $= 32{\cdot}2$ poundals.

METRIC SYSTEM :

The dyne and weight of 1 g.

Weight of 1 g $= 981$ dynes.

TRAINING COURSE

GROUND DUTIES CADETS

Science **2/2b/1**
***2/2b/2**

NOTE :—Paragraphs and Exercises marked thus (*) are to be left until Stage II. (2/2b/2 Science.)

Notes.—(1) Lectures 1 to 6 should be studied in conjunction with the lectures on the corresponding topics in the Preliminary Course, care being taken to revise thoroughly what is repeated and to learn what is new.

(2) Paragraphs and Exercises marked thus (*) are to be left until Stage II. (2/2b/2 Science).

MEASUREMENT.

Revise Lecture 1, 2/1 Science.

STANDARDS OF MEASUREMENT.

(i) The **fundamental units** in the British and Metric Systems are as follows :—

Systems.	Length.	Mass.	Time.
British (or, F.P.S.)	Foot	Pound	Second
Metric (or, C.G.S.)	Centimetre	Gramme	Second

Note.—1 Foot = 30·48 centimetres.
1 Inch = 2·54 cm.
1 Pound = 453·6 grammes.

The second is identical in both systems.

(ii) The British and Metric Systems of measurement are based on these fundamental units. Other units obtained from fundamental units are termed **derived units.**

(iii) The fundamental units can be defined with reference to certain **standard units,** which have been agreed upon in order to have units that are fixed and definite.

The **standard units** in the two systems are as follows :—

Systems.	Length.	Mass.	Time.
British (F.P.S.)	Yard	Pound	Mean Solar Day
Metric (C.G.S.)	Metre	Kilogramme	Mean Solar Day

Standards of Measurement—*continued.*

The fundamental units and the standard units are related as follows :—

3 feet = 1 yard.

86,400 (i.e., 24 × 60 × 60) seconds = 1 mean solar day.

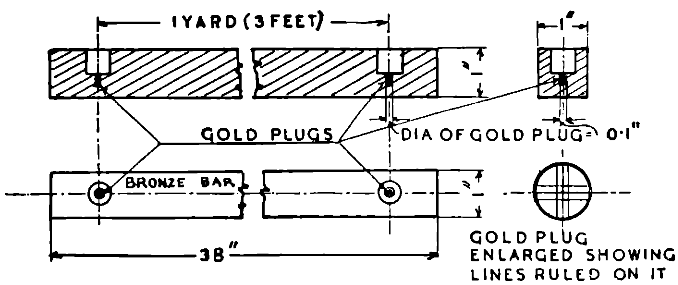
100 centimetres = 1 metre.

1000 grammes = 1 kilogramme.

(iv) The Imperial standard yard is read from the standard bar at a definite temperature, namely 62° F. This is because a bronze bar expands when the temperature rises and contracts when it falls. At temperatures above 62° F., the standard bronze bar will have a length greater than that read at 62° F., whilst at temperatures below 62° F., it will be shorter. The standard bar is mounted on rollers made of bronze. These rollers, placed under the bar in such a way as to reduce possible bending, permit expansion and contraction taking place without distortion.

The temperature, 0° C., at which the standard metre is read, is the temperature of melting ice.

The following diagram illustrates the structure of the Imperial standard yard :—

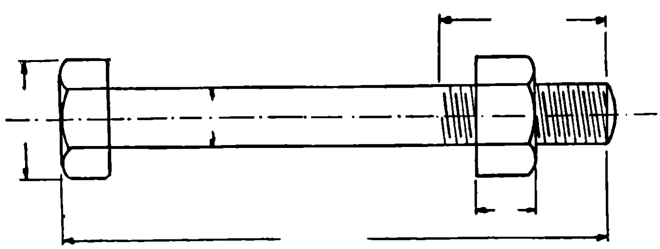


EXERCISES.

1. If the bolt and nut shown below are drawn full size, find, by scaling, the leading dimensions indicated in—

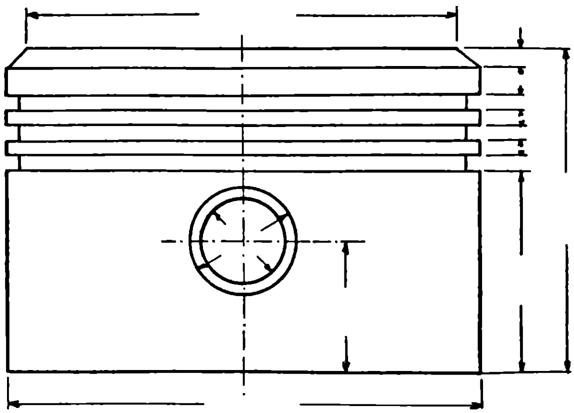
(i) inches and sixteenths.

(ii) millimetres.

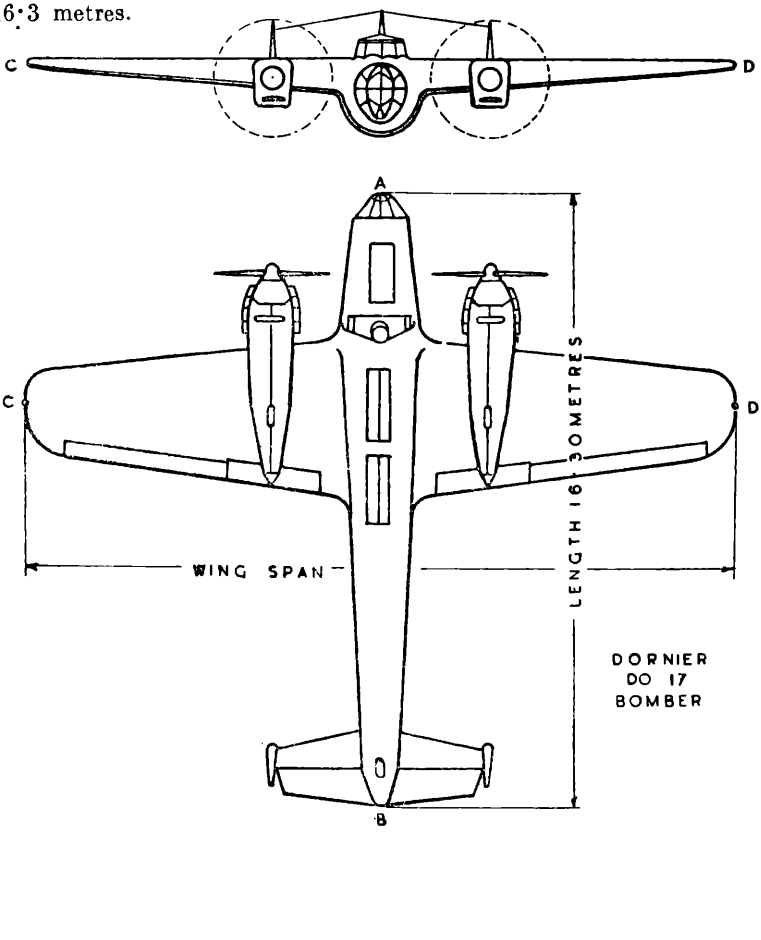


Exercises—*continued.*

2. Find the dimensions indicated on the engine piston shown below, giving each one to the nearest hundredth of an inch (multiply all sizes scaled by 2 to bring to full size).



3. The accompanying diagram represents a German Dornier Do. 17 Bomber monoplane, whose length from A to B is said to be 16·3 metres.



Exercises—*continued.*

(i) Measure the length of AB in the diagram in centimetres.

(ii) State how many metres are represented by 1 cm on the diagram.

(iii) Express the length, 16·3 metres, in feet and inches (to the nearest inch).

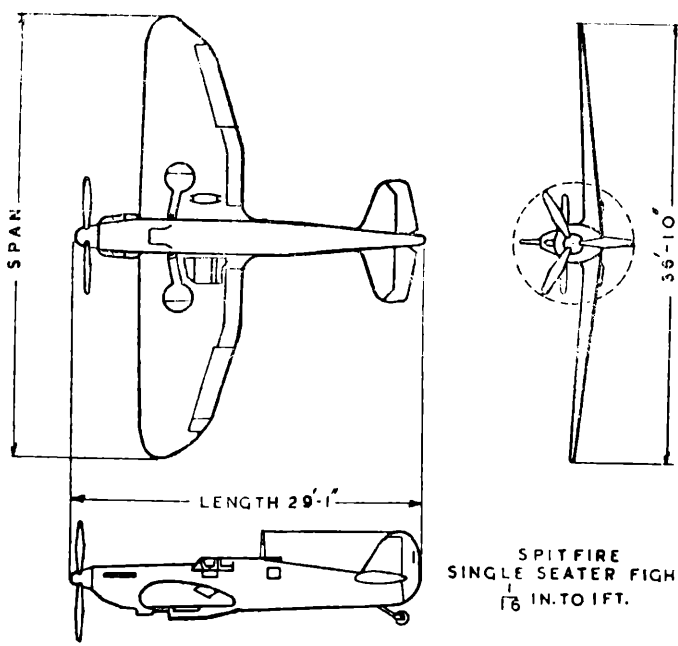
(iv) Find the actual length of the wing span, CD, in metres. Convert this length to feet.

(v) The unloaded weight of the bomber is given as 5,220 kg. Convert this weight to lb. (to the nearest lb.).

*4. Useful work on recognition of aircraft can be combined with this work on measurement, by drawing diagrams of aircraft to an enlarged scale. The shapes may be transferred, by scale-drawing, to cardboard for cutting out.

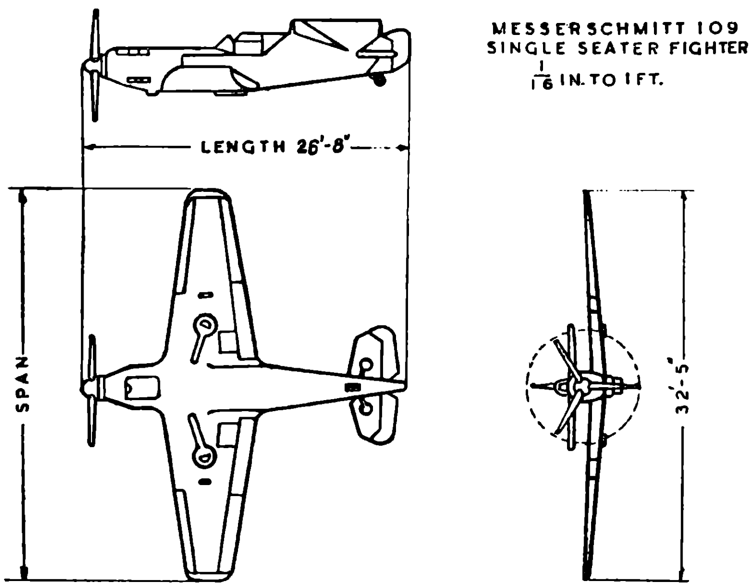
The Dornier Do. 17, drawn in Exercise 3 above, can be used for such purposes.

The following are additional silhouettes that could also be used.



SPITFIRE
SINGLE SEATER FIGHTER
$\frac{1}{16}$ IN. TO 1 FT.

Exercises—*continued.*

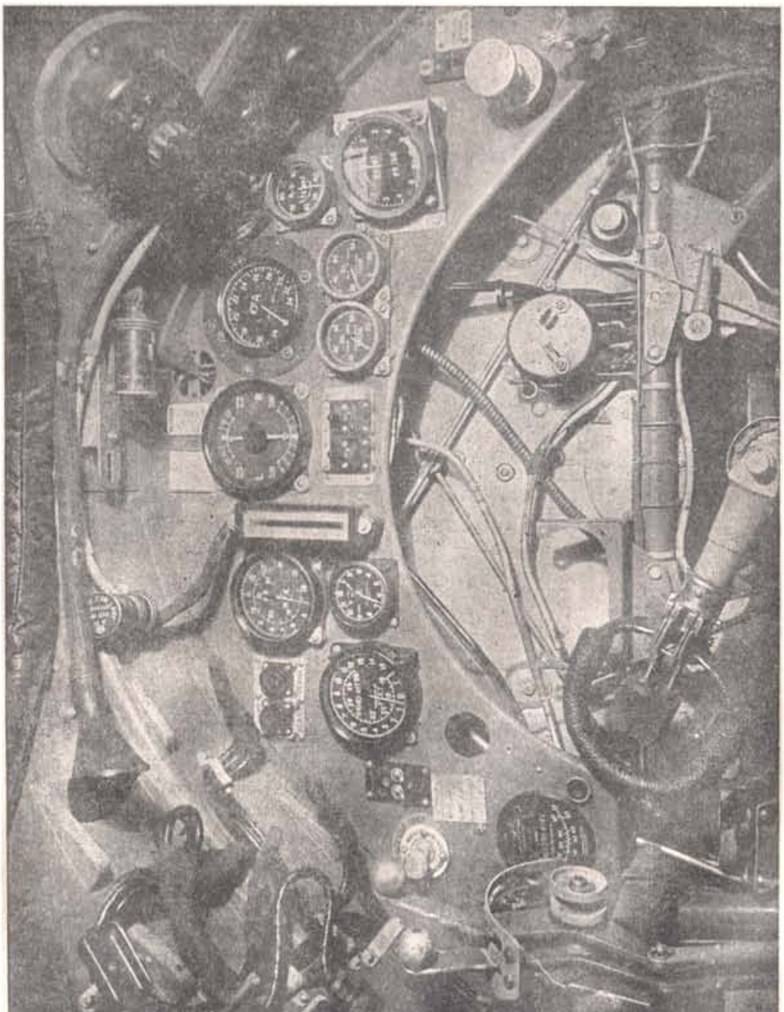


THE LINEAR SCALE.

Revise Lecture 1, 2/1 Science.

It is important to cultivate careful habits in the use of scales as early as possible. Airmen, during their training and whilst on active service, have to use many scales, ranging from the ordinary linear scale for length measurements to various types of computers and instrument scales. An illustration of an instrument panel is showing in the following photograph.

Instruction will be confined in these early stages to the use of linear scales. The rules for the use of linear scales set out in the Preliminary Course should be carefully revised.

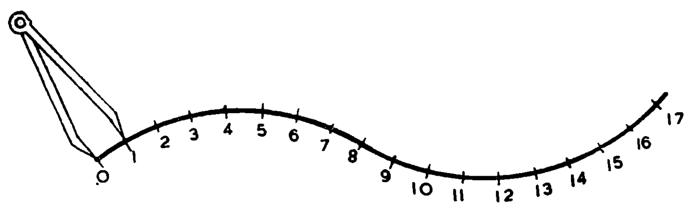


An Instrument Panel

The Linear Scale—*continued.*

(1) Measurement of Curved Lines.

The length of a curved line can be regarded as the sum of the lengths of a large number of small portions, which, because of their shortness, may be regarded as approximating straight lines. The more sections taken, the more nearly does the sum of their lengths approximate to the length of the curved line.



Dividers can be employed for determining the sum of the lengths of the small portions taken. The procedure is as follows:—

(i) One point of the dividers is placed at one extremity of the curved line.

(ii) The dividers are then opened out to span the first portion of the curved line.

(iii) The dividers are rotated slightly on the other point of the dividers until the span is in the same straight line as the second portion of the curve.

(iv) The first point of the dividers is then held firmly in its new position and the span is increased to include the second portion of the curve.

(v) In this way the whole of the curved line is traversed.

(vi) The opened dividers are transferred to a scale and the span is read off. The length of the span is the length of the curved line. If the curved line were too long for one full span of the dividers, the total measurement would have to be taken in stages and the results for all the stages added.

An alternate method for finding the length of the curved line is to take a piece of cotton and knot it at one end. The knot is placed at one extremity of the curved line and the cotton is carefully made to follow the line. The other extremity of the line, when reached, is carefully noted on the piece of cotton. The length of the amount of cotton used gives the length of the curved line.

(2) Checking of Readings.

The lengths of curved lines are difficult to measure very accurately. Consequently, it is advisable to make a series of readings, whatever method is used, and determine the average of the series. A set of readings is given here by way of illustration. If it were stated that the readings were made to the nearest tenth

of an inch, all that could be said from the results is that the length of the curved line does not exceed 2·6 inches and it is not less than 2·4 inches.

Reading 1 :—2·5 in.
2 :—2·4 in.
3 :—2·6 in.
4 :—2·5 in.

$$\text{Average} :— \frac{10\cdot 0}{4},$$

2·5 in.

The mean (or average) value, 2·5 inches, lies between these two extreme values and differs from them by only 0·1 inch, above or below the average.

(3) **Lengths of Curved Lines on Maps.**

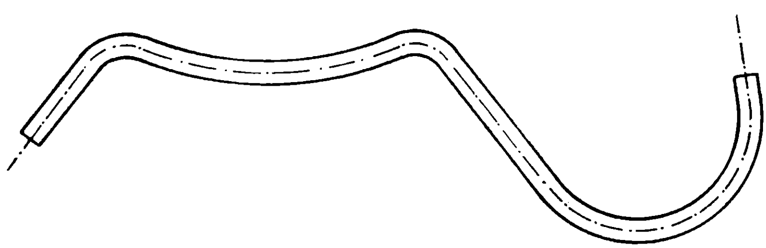
The lengths of curved roads, railway lines or streams, as shown on a map, may be read off in the manner just described.

EXERCISES.

1. A strip of aluminium 3 inches wide and $\frac{1}{8}$ in. thick is to be bent to the shape shown. Find :—

(i) the length of strip required (by scaling).

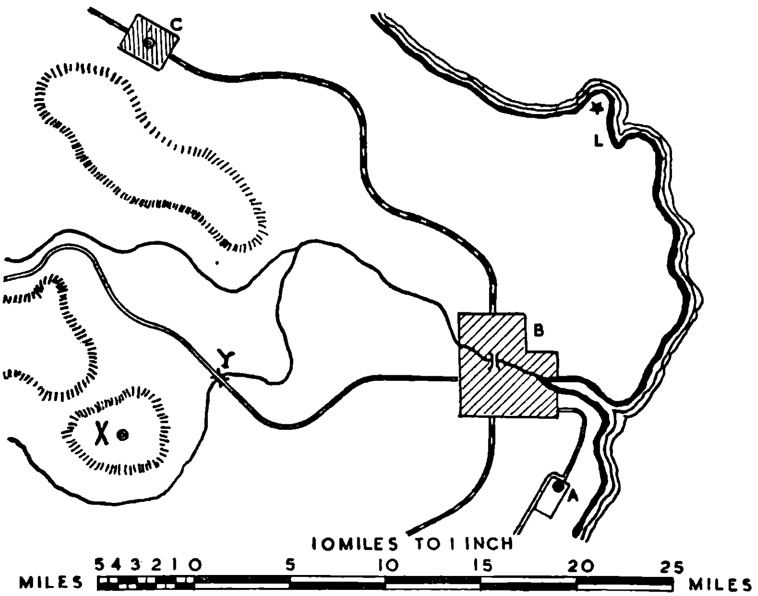
(ii) the weight to the nearest tenth of an ounce. (Density of aluminium : 0·1 lb. per cu. inch.)



2. Draw a circle of radius 3 inches. Measure its circumference by scaling with dividers. Check your result by calculation. What is the ratio of circumference to diameter called ?

Exercises—*continued*.

3. Using the scale indicated at the bottom of the map—



(i) Measure the horizontal distance on the map from the summit of the hill X to the lighthouse L and convert the length to miles.

(ii) What is the length in miles by rail from the boundary of the township C to the boundary of the town B ?

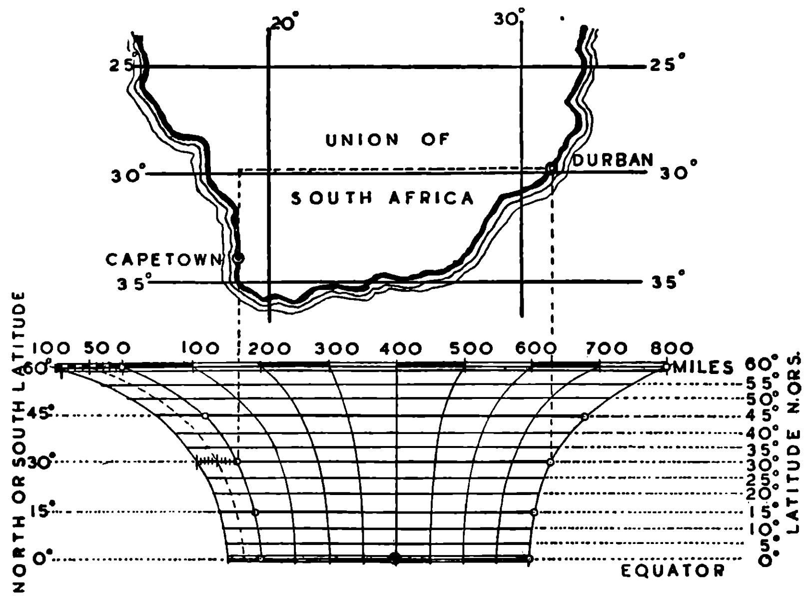
(iii) How many miles would a man have to row from the bridge Y to the point where the stream enters the sea ?

(iv) How far in miles would an aircraft have to fly from directly over the township C to the aerodrome A ?

Exercises—*continued.*

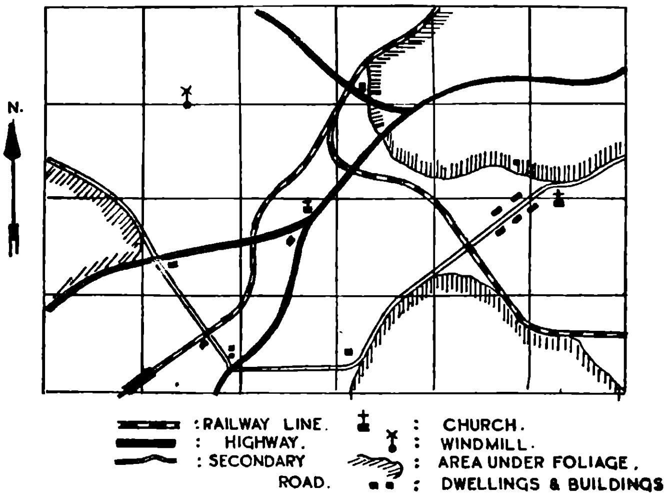
*4. In representing the spherical surface of the earth on a flat surface, various methods of drawing, known as projections, are employed. The projection of the accompanying section of a map is such that a different scale must be employed for measuring lengths at different latitudes. At the equator, the bottom line of the accompanying scale diagram is used; for converting distances on the map to actual distances at latitude 60°, the top line is employed and for intermediate latitudes, the intermediate scale lines.

Measure the distance in a direct line from Capetown to Durban, to the nearest hundred miles.



*5. A good map of your district is particularly useful for gaining familiarity with maps. It will prove useful not only for exercises on this lecture but also for others to come. Distances between prominent landmarks can be measured on the map and, if hikes are arranged, the distances to be covered could be estimated from the map beforehand, whether along straight roads or along curving coastlines and streams.

*6. Find, from the map below, the total lengths of railway lines, highways, and secondary roads. (Scale : 4 miles 1 inch.)



POSITION, DISPLACEMENT, VELOCITY, ACCELERATION.

Revise Lectures 2 and 3, 2/1 Science.

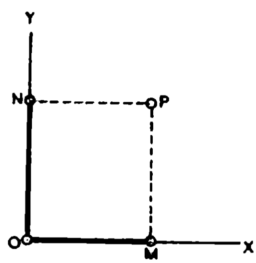
1. POSITION.

(i) The position of a body can be given only in relation to other arbitrarily chosen points or lines of reference.

The following diagrams summarize some important methods for fixing the position of a point.

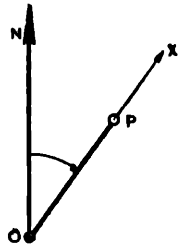
(*a*) Two Given Distances.

The lines OX and OY are at right angles. The position of the point P is fixed by the perpendicular distances PN and PM from OY and OX respectively. Since $OM = NP$, it would be sufficient to measure the distance OM along OX, followed by the measurement of the perpendicular distance MP.



(*b*) A Given Length and a Given Angle.

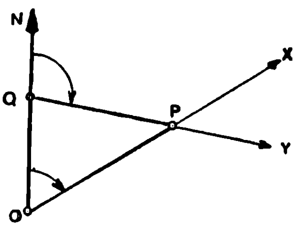
The point P is definitely fixed if we know that it lies along a line OX, which makes a given angle NOX with a given line ON, and is also at a specified distance OP from the point O.



1. **Position**—*continued.*

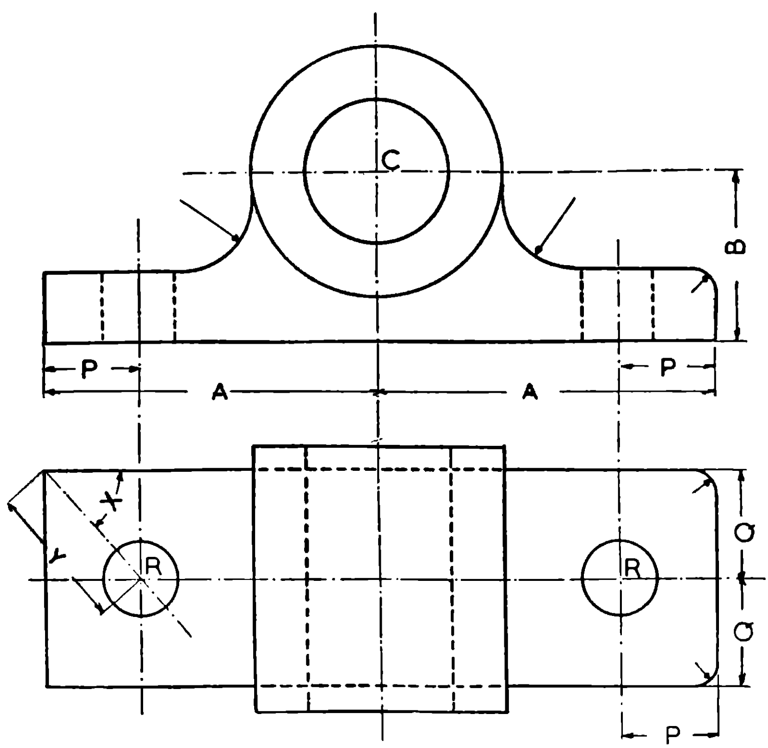
(*c*) **Two Given Angles.**

In this case the point P is fixed if we know it lies along a line OX, making an angle NOX with a given line ON, and also lies on a line QY which makes an angle NQY with ON. It is located at the intersection of the two lines OX and QY.



(ii) **Fixing Position of a hole or centre in a machine part.**

The position of a hole, whether drilled, bored, cored, or otherwise manufactured is most usually fixed by method (*a*) above, using two perpendicular distances from machined or other definite surfaces which are at right angles.



1. **Position**—*continued.*

Method (*b*) using an angle and distance from a point is seldom employed, since it is usually impossible to centre a protractor for marking out, and linear measurements are much more accurate than angular measurements. Method (*c*) is never used at all in Engineering work.

The Solid Bearing shown above will illustrate procedure in marking positions.

Centre C is marked by measurements A and B.

Centre R is marked by measurements P and Q.

The alternate method of fixing R by the angular measurement $X°$ and distance Y is never used, for the reasons given above.

2. MOTION, VELOCITY, SPEED, ACCELERATION.

The notes on speed, velocity, and acceleration, given in the Preliminary Course, should be carefully revised.

The following points should be particularly observed.

(i) When a body changes its position in relation to arbitrarily chosen points or lines of reference, it is said to be in **motion.**

(ii) In changing its position, a body will undergo a **displacement.**

To fully specify a displacement, its magnitude, the direction in which it takes place, as well as its sign, must be stated.

(iii) The **velocity** of a moving body is the rate of its displacement with respect to time.

To fully specify a velocity, its magnitude, direction and sign must be given also.

(iv) The **speed** of a body implies only the rate at which its position is changing, without any stipulation as to direction.

The speed of a body may be constant, but its velocity is changing, if the direction of motion is altering.

(v) The rule for calculating the **magnitude** of velocity or speed is :—

$$\text{Velocity (or speed)} = \frac{\text{distance traversed}}{\text{time taken}}$$

The **units** used for the magnitude of velocity or speed are :—miles per hour, kilometres per hour, feet per sec., &c.

2. Motion, Velocity, Speed, Acceleration—*continued.*

(vi) **Acceleration** is the rate of change of velocity with respect to time.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

To fully specify an acceleration, it is necessary to know its magnitude, direction and sign.

The **units** used for acceleration are usually ft. per sec. per sec. and cm per sec. per sec. but could be m.p.h. per hour, &c. It is important to note that, if a moving body, whose speed is constant, changes direction, it is said to be accelerated. Thus a body, moving in a circle with uniform speed, has an accelerated motion, because its direction is continually changing.

(vii) The quantities, distance, time, acceleration, &c., can all be represented by symbols, which in turn, can be employed to show, concisely, relations between the various quantities, in the form of equations. Those equations of motion already mentioned in the Preliminary Course should be revised, viz. :—

$$a = \frac{v-u}{t}, \quad a = \frac{v}{t}, \quad s = \frac{u+v}{2}t, \quad s = \frac{v}{2}t.$$

EXERCISES.

1. Draw a bolt circle of radius 2 inches, and on it mark off 6 equidistant bolt hole centres. How is this done ?

2. Draw the rectangular base of a machine part, $5\frac{1}{4}$ in. by $2\frac{1}{4}$ in., and on it locate 4 corner holes which are set in $\frac{3}{4}$ in. from the longer sides, and $\frac{7}{8}$ in. from the shorter sides.

3. Discuss the problem of setting out these holes by angular measurements and distances from corners.

4. A body starts from rest with an acceleration of 5 ft./sec.^{2}

Find its velocity at the end of 10 seconds. What distance will it have travelled in that time ?

5. A body is projected vertically upwards with a velocity of 160 ft./sec.

How long will it take to reach its highest point, taking $g = 32$ ft./sec.^{2} ? To what height will it travel ? What will its velocity be at the end of the fourth and the eighth second ?

*6. The rotor of a generator changes in speed from 1,100 r.p.m. to 1,800 r.p.m. in 5 seconds. What is its average angular acceleration ? (r.p.m. = revolutions per minute.)

Exercises—*continued.*

*7. The r.p.m. of an engine is found to vary periodically by $1\frac{1}{2}$ per cent. above and below its mean value of 2,250 r.p.m. If the time between the maximum and minimum speeds is 15 minutes, find the average acceleration and retardation of this engine.

Is this desirable ? How can the engine be regulated ?

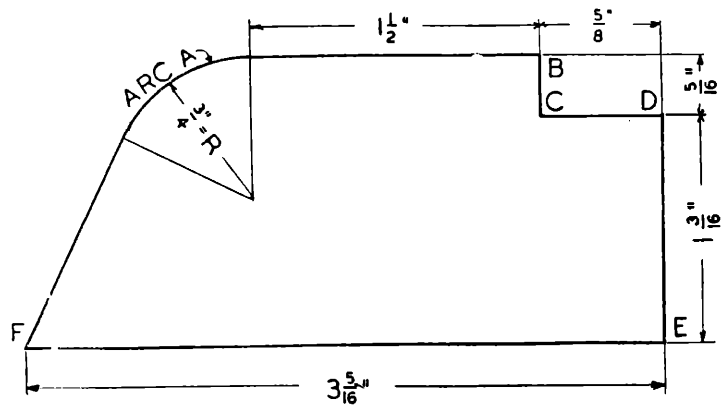
*8. A drilling machine has a speed of 700 r.p.m., and on entering a steel plate its speed is reduced by 1 per cent. Calculate the time it takes to settle down to a constant cutting speed if its average retardation is 1·4 r.p.m. per second.

*9. Draw the gusset plate *ABCDEF* shown below and locate the following bolt holes :—

(i) hole at the centre of arc *A* ;

(ii) 4 holes at $\frac{3}{4}$ inch intervals on a line $\frac{1}{4}$ inch from edge *FE*, the right-hand hole being $\frac{5}{16}$ inch from edge *DE* ; and

(iii) hole $\frac{3}{8}$ inch from top edge *AB* and $\frac{5}{8}$ inch from corner *B*.



Comment on method of location given in (iii) above, and show the correct dimensions.

INERTIA, MASS, FORCE, GRAVITY.

REVISION OF TERMS DISCUSSED IN THE PRELIMINARY COURSE, LECTURES 4 AND 5, 2/1 SCIENCE.

(i) That property of a body by virtue of which it tends to remain in its state of rest, or of uniform motion in a straight line, is termed **inertia.**

(ii) The **mass** of a body is the quantity of matter it contains. It is the mass of a body that causes it to possess inertia.

(iii) A **force** is that which changes, or tends to change, the state of rest of a body or its uniform motion in a straight line. A body cannot of itself change its state of rest or uniform straight line motion. As **Newton's first law of motion** states "Every body continues in its state of rest or of uniform motion in a straight line, unless compelled by external forces to change that state."

(iv) A force of attraction exists between any two masses in the Universe. **Newton's law of Universal Gravitation** states, that "Any two masses attract one another with a force that is proportional to the product of their masses and inversely proportional to the square of their distance apart."

The gravitational force exerted by the earth on a body is termed the **weight** of the body.

It is the force of attraction between the sun and the earth that constrains the latter to move in its orbit. Just as the athlete referred to in the Preliminary Course exerts a force inwards along the handle of the hammer, which he is whirling preparatory to throwing it, so does the sun exert a gravitational force on the earth inwards along a line joining it to the earth. It should be remembered that the orbit of the earth is not a true circle, but an ellipse.

(v) A body will move with uniform velocity so long as a force exists sufficiently large to just overcome forces tending to resist its motion. If no such force exists, or if the one existing is too small, the body will slow down, i.e., undergo **a retardation.** If the force is larger than that just necessary to maintain uniform velocity, the body will gain velocity and so undergo an **acceleration.**

(vi) It is the force of gravity that produces an acceleration in a freely falling body, or a retardation in a body projected upwards ($g = 32 \cdot 2$ ft./sec.^{2} or 981 cm/sec.^{2}).

(vii) If a body falls from a sufficiently great height, the velocity acquired does not increase indefinitely. The **terminal velocity** of the body is that velocity at which the weight of the body and the air resistance are equal. (The air resistance increases as the velocity of the falling body increases, and eventually equals the weight of the body.)

(viii) **From Newton's second law of motion,** a relation between the acceleration produced in a given mass and the force causing it may be established.

Revision of Terms Discussed in the Preliminary Course—*continued.*

This relation may be expressed in the form of an equation,

$$\text{Force} = \text{Mass} \times \text{acceleration}.$$

$$F = m.a.$$

where F is measured in **poundals** (British System) or **dynes** (Metric System). Poundals and dynes may be converted to the other units of force known as **lb. wt.** or **gm wt.** by the relations :—

(*a*) A force of 1 lb. wt. = 32·2 poundals.

(*b*) A force of 1 gm wt. = 981 dynes.

(ix) Newton's third law of motion states "To every force (or action) there is an equal and opposite force (or re-action)."

(x) The force which is the weight of a body may be said to act through the centre of gravity of the body.

EXERCISES.

1. If a stone weighing 2 lb. is thrown with an initial acceleration of 40 ft./sec.^{2}, what force was exerted to do this ?

2. A 3·7 inch anti-aircraft gun fires a shell weighing 24 lb. with an initial velocity of 2,600 ft./sec. If the time of flight to 15,000 feet is 8 seconds, find :—

   (i) the average retardation ;
   (ii) the force to give that retardation ;
   (iii) the pressure on the frontal area of the shell in flight.

3. A force of 100 lb. is exerted by an airman on a bomb trolley carrying two 250 lb. bombs and acts in the direction of motion. Assuming that the resistance to motion is zero, what acceleration would the trolley have ?

Give some comments on your results and the conditions occurring in practice.

*ADDITIONAL EXERCISES.

A useful study of many of the terms given above may be made in relation to a motor car. The following are some leading questions :—

(i) Why is a car started in bottom gear ?

(ii) When one slips the gear into "top," the sensation of a straining by the engine is ordinarily reduced. Why ?

(iii) When the car is braked suddenly, what effect does this have on a passenger ?

(iv) When a car, travelling at a fairly fast rate, rounds a curve, say, to the left, what are the effects on a passenger ?

Additional Exercises—*continued.*

(v) If a level stretch of road is procurable, adjust the throttle until the car is travelling with uniform speed. Notice how long it takes to pass two chosen points and so estimate their distance apart.

If the throttle is gradually opened while travelling over a convenient distance, the change in speed and the time taken for the change can be observed. The acceleration for the interval of time concerned can thus be calculated.

(vi) Note how the pressure of the current of air past the car varies, by holding a hand over the side, from the time when the car is just moving until it is travelling at, say, 40 m.p.h.

(vii) If, on opening the throttle, you found you were still keeping the speed of a car constant, what conclusion could you come to, about the road ?

INERTIA, MASS, FORCE, GRAVITY.

SOME ADDITIONAL APPLICATIONS OF THESE PRINCIPLES.

A fact which Air Force trainees cannot appreciate too early is that the science of Flight is based upon the same fundamental principles as other branches of Mechanics. The following are a few examples which illustrate this fact:—

(i) The Atmosphere.

Flight is impossible in a vacuum. Since the atmosphere is the medium to which a pilot entrusts himself whilst in flight, emphasis will be laid at various stages in these Courses on the properties of the atmosphere. In common with other substances, it possesses mass, it has inertia, it can have a force exerted on it, it can itself exert a force, and so on.

At the outset the mass of the air will be discussed. At later stages and in other Courses, other properties will be considered.

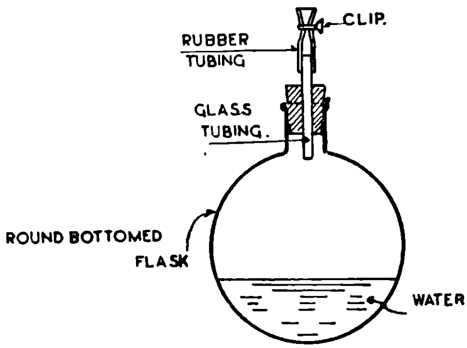
The mass of air. Many people, if they were asked to judge the mass of the air in a room, would reply "almost nothing." Actually, for a room measuring 16 ft. x 14 ft. x 12 ft., the mass of air in it is over 200 lb., whilst in the case of a hall measuring 60 ft. x 30 ft. x 20 ft., the mass is almost 3,000 lb., i.e., well over 1 ton.

The mass of 1 cubic foot of air at sea level is approximately 0·08 lb., i.e., about 1¼ oz. This value becomes less the higher we go in the atmosphere. At 20,000 feet, 1 cubic foot of air has a mass of only 0·04 lb., or about half the value at ground level. Even at sea level the value may alter from place to place, according to climatic conditions, but of this more shall be said later.

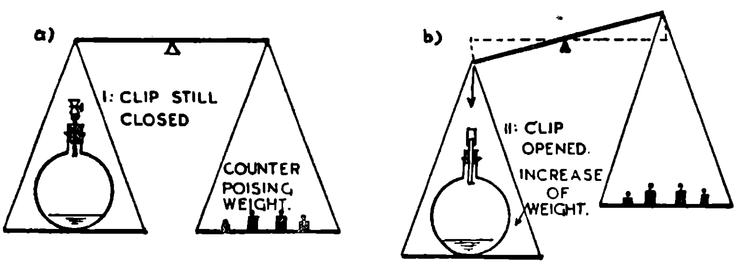
That air has mass can be shown by a simple experiment. A round-bottomed flask of about 250 cm^3 capacity is taken and into it is poured, say, about 50 cm^3 of water. The flask is then fitted with a rubber-stopper, tubing and clip, as shown in the diagram and the water is boiled for a few minutes. The air

Some Additional Applications of these Principles—*continued.*

inside the flask is driven out by the steam. Immediately the flame is removed, the clip should be closed.



After the flask has been cooled to allow the steam within to condense, it can be counterpoised on a balance. If the clip is then opened, the air will enter the flask and an increase in mass will be shown on the balance.



(ii) **Forces acting on an Aircraft.**

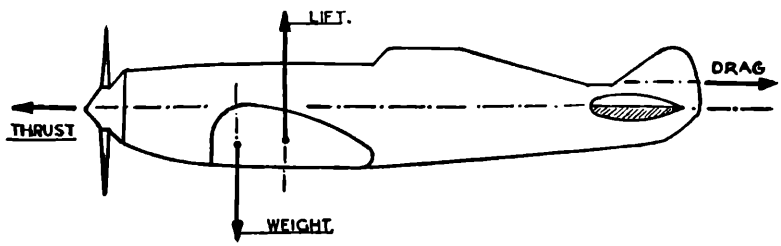
An aeroplane is kept up in the air and travels through it by means of the various forces acting on it. All these forces with the exception of the **weight** of the aircraft, are produced through the medium of the air.

Consider an aircraft in uniform horizontal flight. The fundamental forces acting on it are shown in the diagram overleaf.

(*a*) The motion forward is provided by the **thrust** of the propeller, which usually acts horizontally forwards (although sometimes it is slightly inclined to the horizontal).

Some Additional Applications of these Principles—*continued.*

(*b*) The **drag,** acting horizontally backwards, opposes the motion of the aircraft. Drag is due to the resistance offered by the air to all exposed parts of the moving aircraft. The aim of a designer is to reduce drag to a minimum.



The exposed parts of the aircraft are streamlined, wheels are made retractable, and so on, with this end in view. It is not practicable to eliminate drag entirely.

N.B.—The thrust and drag are equal in magnitude, but opposite in direction.

(*c*) The **weight** of the aeroplane acts vertically downwards.

(*d*) The **lift** for horizontal flight acts vertically upwards, i.e., at right angles to the drag and is equal in magnitude to the weight. The lift is a force produced principally by the movement of the wings through the atmosphere.

The question as to why the thrust should not be greater than the drag is often raised. If the thrust were greater than the drag, the motion of the aircraft would be accelerated; if it were less, the motion would be retarded. Once an aircraft has reached the altitude at which it is intended that uniform horizontal flight be maintained, it is enough for the thrust to be **just** sufficiently large to overcome the drag.

The fact that the lift is equal to the weight in horizontal flight may raise a query similar to that for thrust and drag. If the lift exceeded the weight, the aircraft would climb; if it were less, the aircraft would descend.

Some Additional Applications of these Principles—*continued.*

Should the thrust of the propeller of an aircraft be increased by opening the throttle of the engine, the motion of the aircraft would be accelerated. It can be shown that the lift will increase with an increase in speed. The lift would now be greater than the weight of the aircraft, and so the aircraft would climb to a new level of flight, unless the pilot exercised his controls to maintain the aircraft with its increased speed at the original level. If the thrust be decreased, the lift decreases and the aircraft descends.

A more comprehensive account of the forces acting on an aircraft will be found in the course of technical studies.

EXERCISES.

1. At sea-level the mass of 1 c. ft. of air is approximately 0·08 lb., whereas, at 10,000 ft. above sea-level, the value is approximately 0·06 lb. Find the difference between the mass of the air in a room measuring 20′ x 14′ x 10′ at sea-level and what it would be at 10,000 feet.

2. The lift on an aircraft in uniform horizontal flight was 6,353 lb. wt. Convert the value of this force to poundals.

3. The mass of a train is 250 tons and the resistances, due to friction and air resistance, amount to 15 lb. wt. for each ton. Find the force necessary to maintain a uniform velocity along a level stretch of track. Convert the force to poundals.

*4. A flying boat weighing 4 tons is in uniform horizontal flight. If the lift is 5 times as large as the drag, find what thrust, in lb. wt., the propeller must exert to maintain uniform horizontal flight.

*5. If the train referred to in Exercise (3) underwent a uniform retardation of 3·2 ft./sec.^{2}, what would be the force acting, expressed in lb. wt. ?

*6. If the flying boat referred to in Exercise (4) underwent a uniform acceleration of 5 ft./sec.^{2} without change in drag, what thrust is the propeller now exerting ?

CENTRE OF GRAVITY—UNITS OF FORCE.

1. Revise Lecture 6, 2/1 Science.

2. Revision will comprise work on **Centre** of **gravity** and methods for determining its position.

REVISION EXERCISES.

1. Cut model aircraft out of heavy cardboard using the silhouettes given in Lecture 1, 2/2b/1 Science. Find the centre of gravity of each model. What assumption is made if this is taken in each case as the **actual** centre of gravity ?

2. Draw up a table summarizing the positions of centres of gravity for various regular geometrical figures. This table will be of use in future work.

3. What would be the effect on the centre of gravity of a bomber if the rear gunner came forward ? Where would the bombs be carried ? Give your ideas on what effect dropping the bombs would have on the aircraft.

*4. Where would you think the combined centre of gravity of a parachute trooper and his outfit would be—

(i) before jumping from the troop carrier ;

(ii) just before landing ?

Does your answer suggest to you anything about the trooper's stability just before landing ? (**N.B.**—Assume the parachute has opened.)

*5. Where should the centre of gravity of all **rotating** parts lie ? Discuss the problem with reference to rotation of propellers, crankshafts, electric motors and generators, power driven tools.

DENSITY AND SPECIFIC GRAVITY.

1. DENSITY.

(1) It is common in ordinary conversation to hear one substance spoken of as "heavier" than another. The comparison really depends, of course, on the volume of each substance considered.

Thus, whilst the mass of 1 **cubic foot** of aluminium is slightly less than one-quarter that of a **cubic foot** of lead, it is 400 times greater than that of a **cubic inch** of lead.

Comparisons of this kind should be made only by considering masses of equal volumes of the two substances concerned. In comparing solids and liquids, the units of volume usually employed are the cubic foot, in the British system, and the cubic centimetre, in the Metric system. For gases, multiples of the cubic foot, the cubic metre and the litre are generally employed, e.g., the gas capacity of the ill-fated R101 airship was given as $5\frac{1}{2}$ million cubic feet.

(2) The mass per unit volume of any substance is called its **density.**

Thus, the density of aluminium is approximately 165 lb. per cubic foot or $2 \cdot 7$ g per cm^3, whilst that of lead is 710 lb. per cubic foot or $11 \cdot 4$ g per cm^3.

(3) The mass per unit volume (or density) of any substance may be calculated, if the mass of **any** volume of it is known.

EXAMPLE.

If 50 cm^3 of gold were found to have a mass of 965 g, what is the density of gold ?

The mass of 50 cm^3 of gold is 965 g.

$$\text{The mass of } 1 \text{ cm}^3 \text{ of gold is } \frac{965}{50} \text{ g,}$$
$$= 19 \cdot 3 \text{ g}$$

i.e. the density of gold is **19·3 g per cm^3.**

(4) Knowing the mass of **unit** volume of any substance it is a simple matter to find the mass of other volumes of it.

EXAMPLE.

Find the mass of 100 cm^3 of mercury, given that the density of mercury is $13 \cdot 6$ g per cm^3.

The mass of 1 cm^3 of mercury is $13 \cdot 6$ g.
The mass of 100 cm^3 of mercury is $(13 \cdot 6 \times 100)$ g
$= \mathbf{1360}$ **g.**

(5) If the density of a substance is known, the volume of any mass of it can be calculated.

EXAMPLE.

Find the volume of 1,248 lb. of water, given the density of water as $62 \cdot 4$ lb. per cubic foot.

$62 \cdot 4$ lb. of water have a volume of 1 c. ft.

$$\therefore \text{ 1248 lb. of water have a volume of } \frac{1}{62 \cdot 4} \times 1248$$
$$= 20 \text{ c. ft.}$$

1. **Density**—*continued.*

*(6) To find the density of any substance experimentally, it is necessary to ascertain both its mass and volume. The method used for finding these two values will depend upon the nature of the substance, i.e. whether it is a solid, a liquid or a gas. The examples described hereunder are meant to act as guides for work that may be performed in class, if apparatus is available.

Example 1.

To find the density of a **regular solid,** say, a cube of brass.

Procedure.

(1) Measure the length of a side of the cube. Suppose it is 2 cm.

(2) Calculate the **volume** of the cube. The volume of the cube $= 2 \times 2 \times 2 = 8$ cm^3.

(3) Find the **mass** of the cube on a balance. Suppose it is 68 gm.

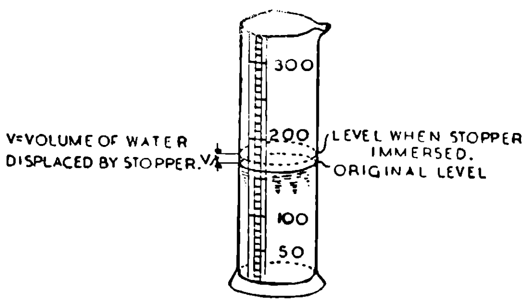
Since the mass of 8 cm^3 of brass $= 68$ g,

$$\text{the mass of 1 cm}^3 \text{ of brass} = \frac{68}{8}\text{ g} = 8{\cdot}5\text{ g}.$$

i.e. the density of the brass concerned is $8{\cdot}5$ g per cm^3.

Example 2.

To find the density of **an irregular solid,** say, a glass stopper.



Procedure.

(1) Find the **volume** of the stopper, by immersing it in a known volume of water in a graduated cylinder.

The stopper will displace its own volume of water. The level of the water will rise in the cylinder and the volume of the stopper can be read off from the scale on the cylinder.

Suppose the volume is 10 cm^3.

(2) Find on a balance the **mass** of the stopper when dry.

Suppose it is $25{\cdot}2$ g.

Example 2—*continued.*

Since the mass of 10 cm^3 of glass = 25·2 g,

$$\text{the mass of } 1\ cm^3 \text{ of glass} = \frac{25 \cdot 2}{10}\ g, = 2 \cdot 52\ g.$$

i.e. the density of the glass is approximately 2·5 g per cm^3.

Example 3.

To find the density of a **liquid.**

Procedure.

(1) Counterpoise a glass beaker on a balance.

(2) Run in an accurately measured **volume** of the liquid.

(3) Find the **mass** of the liquid run in.

Suppose 100 cm^3 of **water** were run in, and its mass were found to be 99 g.

Since the mass of 100 cm^3 of water = 99 g,

$$\text{the mass of } 1\ cm^3 \text{ of water} = \frac{99}{100}\ g, = 0 \cdot 99\ g.$$

i.e. the density of water (as found here) = 0·99 g per cm^3
= **1 g per cm³**, approx.

Example 4.

To find the density of a **gas** (say, air).

N.B.—It will be pointed out, in a later lecture, that air is really a **mixture** of gases, but that fact will not alter the following procedure. It was shown earlier that air has mass. The density of air may be determined by employing the same apparatus as used in Lecture 5.

Procedure.

(1) Precisely the same method is used to the stage where air is allowed to enter the flask.

(2) The **mass** of the air that enters is then found, as accurately as possible, by adding weights to restore balance.

(3) The **volume** of the air is found, by measuring both the volume of water left in the flask and the total volume required to fill the flask to the bottom of the stopper. The difference between these two readings gives the volume of air that entered the flask.

Suppose the mass of the air = 0·246 g,
and its volume = 200 cm^3.

The mass of 200 cm^3 of air = 0·246 g,

$$\therefore \text{ The mass of } 1\ cm^3 \text{ of air} = \frac{0 \cdot 246}{200}\ g, = \cdot 00123\ g.$$

Example 4—*continued.*

This result is an exceedingly small value, hence, it is better to state the density as the number of g per litre (1000 cm^3).

The mass of 1000 cm^3 = $\cdot$00123 × 1000 = **1$\cdot$23 g.**

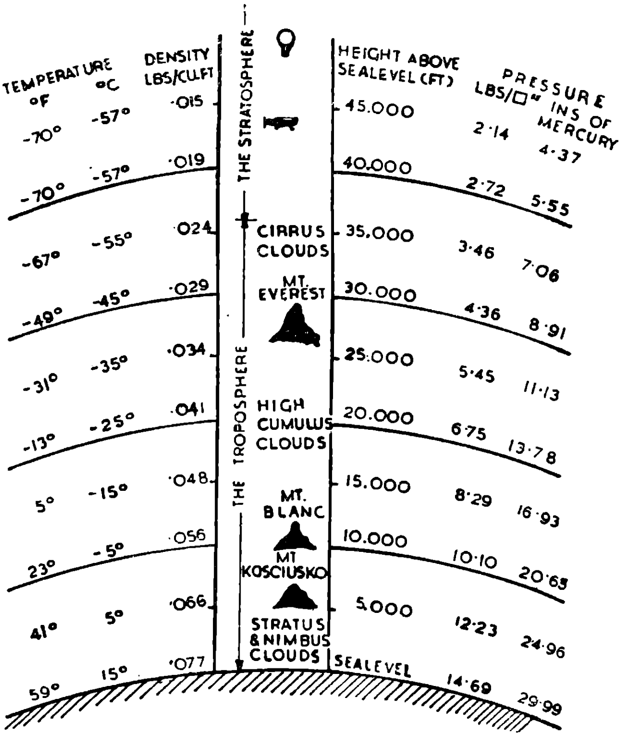
i.e. the density of air, **under the room conditions,** = 1$\cdot$23 g per litre.

Converting to British Units, the density of the air = approximately **0$\cdot$077 lb. per cubic foot.**

N.B.—This latter result should be checked in order to obtain practice in the conversion of units.

*2. DENSITY OF THE ATMOSPHERE.

The density of the atmosphere surrounding the earth is not uniform throughout, but decreases with an increase in altitude. At sea-level, 1 cubic foot of air has a mass of approximately 0$\cdot$08 lb. per cubic foot. The following diagram illustrates the manner in which the density of the atmosphere varies with altitude.



N.B.—At any given locality, the density of the atmosphere varies from day to day, due to changes occurring in temperature and pressure. This variation will be further discussed at a later stage.

***3. DENSITY OF METALS, ETC.**

The following table shows the densities of materials used in aircraft construction :—

Material.	Density in lb./ft.³.	Material.	Density in lb./ft.³.
Lead ..	712	Glass	181
Monel metal	554	Water	62·5
Copper	549	Mahogany (propellers)	53
Brass	506	English Ash	49
Steel	487	Birch (plywood)	44
Tin	456	Spruce ..	30
Zinc	449	Oil (petroleum)	55
Chromium ..	406	Petrol	46
Duralumin (Al. Alloy)	178	Cork	15
Aluminium ..	167		
Elektron (Mag. Alloy)	114		

4. SPECIFIC GRAVITY.

(1) The density of cast iron is approximately 450 lb. per cubic foot. Therefore, the mass of 1 cubic foot of cast iron is about $\frac{450}{62\cdot5} = 7\cdot2$ times that of 1 cubic foot of water.

Similarly, since the density of petrol is approximately 46 lb. per cubic foot, it can be said that the mass of 1 cubic foot of petrol is about $\frac{46}{62\cdot5} = \cdot73$ that of 1 cubic foot of water.

(2) The **specific gravity** of a substance is the **number** obtained by dividing the mass of a given volume of the substance by the mass of an equal volume of water at the same temperature.

Hence, the specific gravity of cast iron is 7·2, whilst that of petrol is 0·73.

The use of the term "specific gravity" is restricted to solids and liquids. Hydrogen is the lightest of all gases and it is with its density that the densities of all other gases are compared.

The value of specific gravity is only a **number.** No units are required. If it is said that the specific gravity of cast iron is 7·2, it means that **any** volume of cast iron has a mass 7·2 times greater than that of an **equal** volume of water at the same temperature.

Thus the mass of 10 c. ft. of cast iron $= 7\cdot2 \times (10 \times 62\cdot5)$ lb.
$=$ **4500 lb.**

Similarly the mass of 10 cm³ of cast iron $= 7\cdot2 \times (10 \times 1)$ g,
$=$ **72 g.**

4. Specific Gravity—*continued.*

NOTE :—The value of the specific gravity of a substance is the same in both the British and the Metric systems, but whereas the density and the specific gravity of a substance have the same numerical values in the Metric system, they are different in the British system. Take cast iron again as an example :

System.	Specific Gravity of Cast Iron.	Density of Cast Iron.
British	7·2	450 lb. per c. ft.
Metric ..	7·2	7·2 g per cm^3

EXAMPLES.

(1) A cube of common glass has sides measuring 2 cm and weighs 20 g. What is the specific gravity (S.G.) of the glass ?

The volume of the cube = $(2 \times 2 \times 2)\ cm^3 = 8\ cm^3$.
8 cm^3 of glass weigh 20 g.
8 cm^3 of water weigh 8 g.

$\therefore$ S.G. of the glass $= \frac{20}{8} = \mathbf{2{\cdot}5}$

(2) What is the density of the glass in Example (1) ?

In the Metric System.

1 cm^3 of glass is 2·5 times heavier than 1 cm^3 of water.

$\therefore$ 1 cm^3 of glass has a mass of $(2{\cdot}5 \times 1)$ g. = **2·5 g.**

i.e. the density of the glass is **2·5 g. per cm^3.**

In the British System.

1 c. ft. of glass is 2·5 times heavier than 1 c. ft. of water.

$\therefore$ 1 c. ft. of glass has a mass of $(2{\cdot}5 \times 62{\cdot}5)$ lb. = 156 lb. approx.

i.e. the density of the glass is **156 lb. per c. ft.**

(3) The specific gravity of petrol is 0·73.

(*a*) Find the mass of 1 c. ft. of petrol.

(*b*) What is the mass of 1 gallon of petrol ?

The mass of 1 c. ft. of petrol = 0·73 × the mass of 1 c. ft. of water.
= $(0{\cdot}73 \times 62{\cdot}5)$ lb.
= 45·6 lb. = **46 lb.** (approx.)

The mass of 1 gallon of petrol = 0·73 × the mass of 1 gallon of water.

i.e. the mass of 1 gallon of petrol = $(0{\cdot}73 \times 10)$ lb.
= **7·3 lb.**

Examples—*continued.*

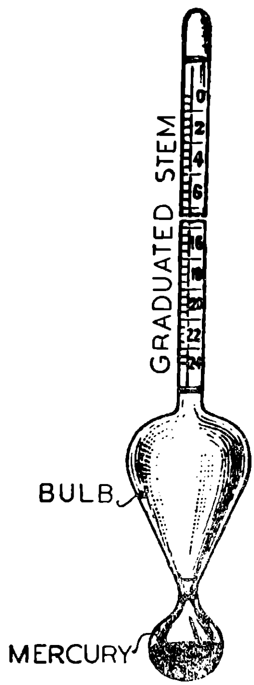
(4) A 10 cm^3 bottle specially used for finding the specific gravity of liquid weighs :—

19·43 g when empty,
29·42 g when full of water,
28·19 g when full of oil.

What is the S.G. of the oil ?

$$\text{The mass of } 10\ cm^3 \text{ of oil} = (28{\cdot}19 - 19{\cdot}43)\ g,$$
$$= 8{\cdot}76\ g.$$
$$\text{The mass of } 10\ cm^3 \text{ of water} = (29{\cdot}42 - 19{\cdot}43)\ g,$$
$$= 9{\cdot}99\ g.$$
$$\text{Hence, the S.G. of the oil} = \frac{8{\cdot}76}{9{\cdot}99} = \mathbf{0{\cdot}877}.$$

*5. **Hydrometers** are instruments used to measure the specific gravity of liquids. A common type is of the form illustrated. The denser the liquid, the higher will the hydrometer ride in the liquid.



A scale is devised and marked on the stem of the hydrometer. From it can be read the Specific Gravities of liquids over a range for which it was manufactured. Hydrometers are often bought in sets to cover a wide range of Specific Gravities. The **Acidometer** is used to measure the Specific Gravity of acids, e.g. that in a battery.

*6. **Classification of aircraft.**—Aircraft may be divided into two broad classes :—

(1) **Aerostats** (lighter-than-air aircraft)

(2) **Aerodynes** (heavier-than-air aircraft).

Balloons and **airships** (rigid, semi-rigid and non-rigid) are aerostats.

The **kite, glider** and **aeroplane** (land, sea and amphibian) are aerodynes.

The lift necessary for the flight of an aerodyne is produced by its motion through the air. It will be the object in the Theory of Flight to explain exactly how this lift is produced. The aerostat is able to rise from the ground since its weight is less than the weight of the air it displaces, and this results in an upthrust occurring on the aerostat.

EXERCISES.

1. Find the density of :—

   (*a*) 50 cm^3 of tin weighing 365 g.
   (*b*) 12 cm^3 of brass weighing 100·8 g.
   (*c*) 20 cm^3 of lead weighing 228 g.
   (*d*) 11 cm^3 of glass weighing 27·5 g.

2. What is the volume of :—

   (*a*) 5 g of sulphur, whose density is 2 g per cm^3.
   (*b*) 2·5 g of cork, whose density is 0·25 g per cm^3.
   (*c*) 81 g of marble, whose density is 2·7 g per cm^3.

3. What will the following weigh :—

   (*a*) 9 cm^3 of ivory, density 2 g per cm^3.
   (*b*) 5 cm^3 of lead, density 11·4 g per cm^3.
   (*c*) 4 cm^3 of glass, density 2·5 g per cm^3.

*4. A rectangular block of soft wood measures 20 in. × 1·5 in. × 14 in. If its mass is 7·28 lb., calculate the density of the wood. What timber do you consider this is ?

*5. Using the density scale given above for the atmosphere, find the mass of 1,000 cubic feet of air at an altitude of 20,000 feet.

*6. Suppose that, at a certain altitude, 27 cubic feet of air weighed 1·512 lb. Calculate the altitude of the air concerned, employing the scale above.

*7. The capacity of the Graf Zeppelin is 3,710,000 cubic feet. What would be the weight of this volume of hydrogen, if 1 cubic foot of hydrogen has a mass 0·0056 lb. ?

Exercises—*continued.*

8. An empty beaker weighs 44 g. 20 cm^3 of turpentine are run in and the beaker and contents weigh 61·6 g. What is the S.G. of turpentine ?

9. The Specific Gravity of ice is 0·918. What is the mass of 2 c. ft. of ice ?

10. The Specific Gravity of sea-water = 1·02. Find the mass of 1 litre of sea-water.

11. A 100 cm^3 flask, full of sulphuric acid whose S.G. is 1·84, weighs 220 g. Find the mass of the flask when empty.

*12. A cube of material has sides measuring 3 inches. If it weighs 2·78 lb., find the density and S.G. of the material. What material do you consider this is ?

*13. The S.G. of the bronze in a penny is 8·96. If 3 new pennies weigh 1 oz. what is the volume in cubic inches of the metal in £10 of new pennies ?

PRESSURE IN FLUIDS.

1. The term pressure has been used in a general sense in previous lectures. It is important that its correct meaning should be understood.

Consider a light flat plate, 4 sq. ft. in area, suspended, as shown, at right angles to a wind, the force exerted on its surface being measured by a suitable spring balance. The main string connected to the spring balance should be centrally adjusted, parallel to the air-flow.

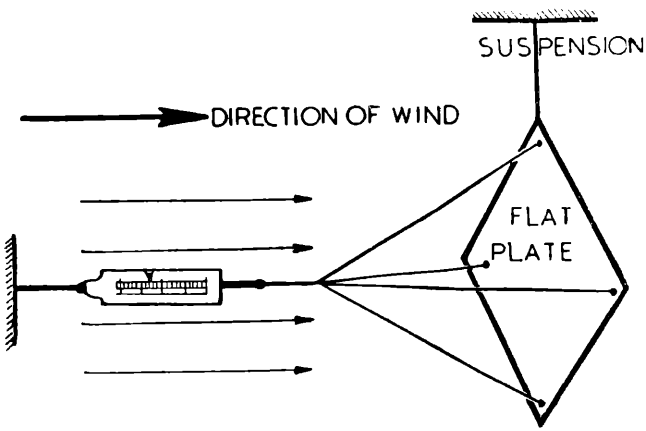


Fig. 1.

Suppose the force registered on the spring balance (or **dynamometer** as it may be called) were 16 lb. wt. Now consider the plate as being replaced by another whose area is 8 sq. ft. It would be seen that the force was greater, say, 32 lb. wt.

The force of the wind measured in this way depends on the area of the plate. For comparison, it is better to determine the force on **unit** area in each case.

In the first case, the force **per sq. ft.** $= \frac{16}{4} = 4$ lb. wt.

In the second case, the force **per sq. ft.** $= \frac{32}{8} = 4$ lb. wt.

Pressure in Fluids—*continued.*

The force of the wind per unit area is termed its **pressure.**

The pressure of the wind is thus 4 lb. wt. per sq. ft. (or $\frac{1}{36}$ lb. per sq. inch.)

In general, **pressure** may be defined as **force per unit area.**

i.e. $\text{Pressure} = \dfrac{\text{Force exerted}}{\text{Area concerned}}$

or, in symbols, $P = \dfrac{F}{A}$

N.B.—In order to perform the above experiment, it may be difficult to obtain, in the open, a wind that is blowing steadily and in a uniform direction. Inside the laboratory, a wind may be produced by an electric fan, the flow being straightened out into a steady stream by placing in front of it a box fitted with a honeycomb of glass tubes. The diagram suggests an arrangement of fan and tubes.

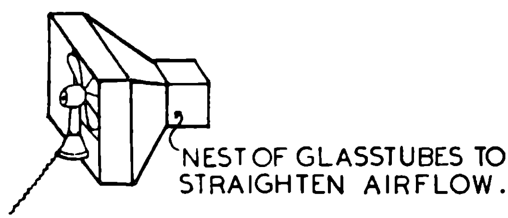


Fig. 2.

2. If the pressure exerted on a surface is known, the total force acting over the whole area of the surface can be calculated. Thus, suppose the pressure, uniformly distributed over a certain plate, were 50 lb. per ft.^{2}, find the total force exerted on the plate, if its area were 700 ft.^{2}

The force on 1 ft.^{2} is 50 lb.; on 700 ft.^{2} it is (50×700) lb. $=$ 35,000 lb. $= 15{\cdot}6$ tons.

The total force on a surface

$=$ **The Pressure acting on it $\times$ the Area of the surface.**

or in symbols, $\mathbf{F} = \mathbf{P} \times \mathbf{A}.$

Pressure in Fluids—*continued.*

*3. A cubic centimetre of lead weighs 11·4 g, and a cubic centimetre of aluminium weighs 2·7 g. Consider two vertical

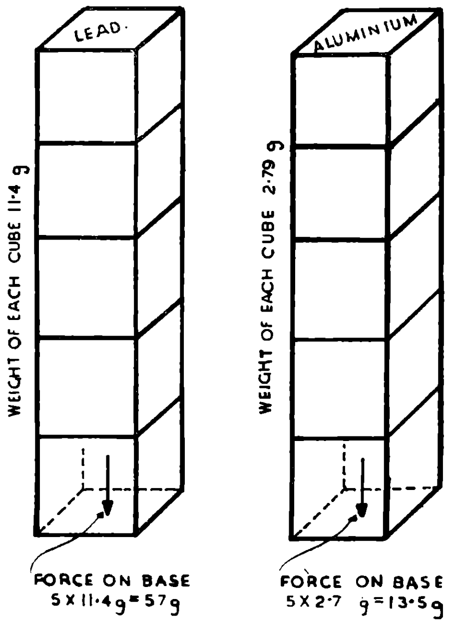


Fig. 3.

columns, one made up of 5 cubes of lead and the other of 5 cubes of aluminium, the volume of each cube being one cubic centimetre.

The weight of the column of lead $= 5 \times 11{\cdot}4 = 57$ g.

The weight of the column of aluminium $= 5 \times 2{\cdot}7 = 13{\cdot}5$ g.

Since the area of the base of each column is 1 cm² the pressure at the base of the column of lead = 57 g per cm²; and the pressure at the base of the column of aluminium = 13·5 g per cm². If the heights of the columns were increased, i.e., more cubes added, the pressure at the base would be increased. If the heights were doubled, the pressures would be doubled, and so on.

*4. **The pressure exerted by each column** depends upon two factors :

(1) The density of the material in the column, and

(2) The height of the column.

The pressure at the base of each column

= its vertical height × the density of the material in it.

or, in symbols, $\mathbf{P} = \mathbf{h} \times \mathbf{d}$.

Pressure in Fluids—*continued.*

5. A pressure exists at the base of a column of liquid. Consider a heavy stand supporting a glass cylinder open at both ends, set up as in the diagram. Suppose a disc is pressed up against the bottom of the cylinder by means of a thread attached to one arm of a balance, as shown.

A weight W is placed in the balance pan attached to the other arm, and water is poured slowly into the cylinder until it is just seen to escape at the disc. As weight W is increased, the height h of water in the cylinder marked by the pointer is found to increase in the **same proportion.**

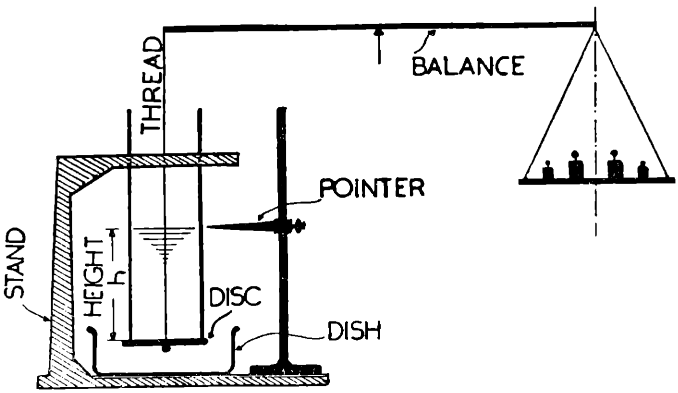


Fig. 4.

Suppose the following readings were made on the apparatus :—

Depth of water in the cylinder = 30 cm.

Area of cross-section of cylinder = 25 cm^2.

Force required to support weight of water = 745 g.

(**N.B.**—Allowance is made for the weight of the disc.)

Then, the force due to the weight of the water acting on 25 cm^2 = 745 g.,

$$\text{or, the force acting on 1 cm}^2 = \frac{745}{25} = \mathbf{29{\cdot}8\ g.}$$

The pressure at the base of the column of water 30 cm high is therefore approximately 30 g per cm^2.

If the height of the column were increased first to 35 cm and then to 40 cm, it should be found that the pressures at the base were 35 g per cm^2 and 40 g per cm^2 respectively.

If the water were replaced by another liquid, say, brine or oil, the pressures for the heights specified above would be greater for the denser liquid and smaller for the less dense.

Pressure in Fluids—*continued.*

Summing up, the pressure at the base of a column of liquid depends upon:

(1) the height of the column,

(2) the density of the liquid.

The value of the pressure is calculated by the **product** of these two quantities, expressed in appropriate units.

The units frequently employed may be summarized:

System.	Pressure.	Height.	Density.
British	lb. per sq. ft.	feet	lb. per c. ft.
Metric	g per cm^2	cm	g per cm^3

The relation for the pressure at the base of a column of liquid may be expressed in symbols as follows:—

$$\mathbf{P = h \times d.}$$

Where P = the pressure
h = the height of the column
d = the density of the liquid.

NOTES.—(1) The pressure at any level **within** a liquid is given similarly by the product of the depth of the level below the surface of the liquid and the density of the liquid.

(2) By using cylinders and discs of different cross-sectional area and filling them with water to the **same** height, thus giving the **same pressure** P, it can be shown that the weight W needed to balance the water in the cylinder is proportional to the cross-sectional area of the cylinder A;

$$\text{or } W = \text{pressure} \times \text{Area}$$
$$= P \times A,$$

as was found in paragraph 2.

N.B.—Allowance is made for the weight of the disc.

6. **Worked examples.**

(1) If the density of sea-water is 1·025 g per cm^3, what is the pressure at a depth of 68 metres?

Required Pressure (in g per cm^2) = depth (in cm) × density of the sea-water (in g per cm^3)
= 6800 × 1·025
= **6970 g per cm^2**
= (6·970 kg per cm^2).

Pressure in Fluids—*continued.*

Employing dynes, the pressure may be expressed as (6970×981) dynes per cm^2.

$$= 6{\cdot}8 \times 10^6 \text{ dynes per cm}^2.$$

(2) What is the pressure at a depth of 5 fathoms in a fresh-water lake? (Take the density of the water as $62\frac{1}{2}$ lb. per c. ft.).

Required Pressure = depth × density of fresh water
(in lb. per sq. ft.) (in feet) (in lb. per c. ft.)

$$= 30 \times 62\tfrac{1}{2}$$
$$= \mathbf{1875\ lb.\ per\ sq.\ ft.}$$

If the pressure were wanted in lb. per sq. inch, this value need only be divided by 144.

*7. Several other features of the pressure in liquids should be noted.

(1) The pressure of a liquid at all points in a horizontal plane is the same and is given by the product of the density of the liquid and the vertical distance between the free surface of the liquid and the plane. Thus in Fig. 5, *AB* is a horizontal plane and the pressure at all points on *AB* is given by

$$P = hd$$

Where P = pressure
h = vertical distance to free surface
d = density of the liquid.

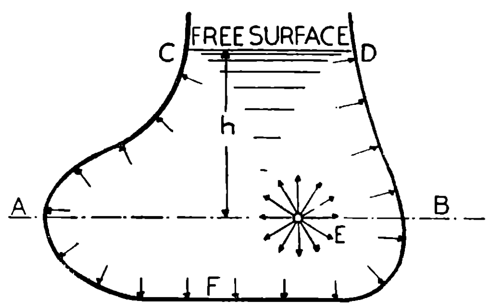


FIG 5

(2) The pressure at any point in a liquid is the same in all directions (e.g. at E in Fig. 5 the pressure is $P = hd$ in every direction).

(3) The pressure exerted by a liquid on a surface with which it is in contact is always perpendicular to that surface (as shown by small arrows on surface *CAFBD* in Fig. 5).

Pressure in Fluids—*continued.*

From the above it may be shown that the pressure on the bottom of a vessel containing a liquid does not depend on the shape of the vessel nor upon the quantity of liquid it contains, but on the depth of the bottom below the free surface only.

For example, consider the three vessels (*a*), (*b*) and (*c*) in Fig. 6.

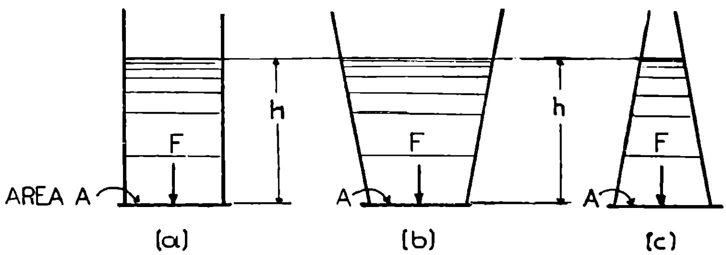


FIG 6

The area, A, of the bottom is the same in each case and consequently the total force F on the bottom is also the same in each case, i.e.,

$$F = PA = hdA$$

Where $P = hd =$ pressure,

$h =$ depth of liquid and

$d =$ density of liquid.

This may be demonstrated by using apparatus as shown in Fig. 4, if the cylinder were replaced by vessels of different shapes.

It may also be demonstrated by using vessels shaped as in Fig. 6 and with loose plates at the bottom and then sinking them to the same depth h in a liquid as shown in Fig. 7.

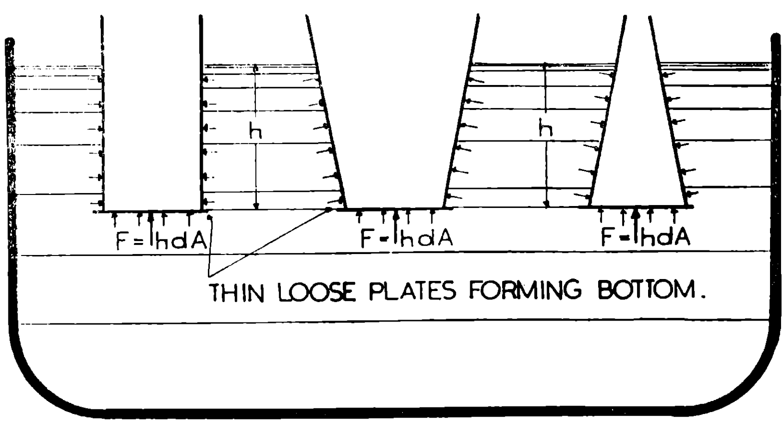


FIG 7

Pressure in Fluids—*continued.*

The liquid presses on the vessels as shown and the total force on the bottom of each vessel is $F = hdA$.

Now if the vessels are carefully filled with liquid it will be found that, as soon as the level inside is the same as that outside, the loose plates forming the bottoms will drop off due to their own weight, thus showing that the total force on the bottom due to the enclosed liquid as in each case $F = hdA$.

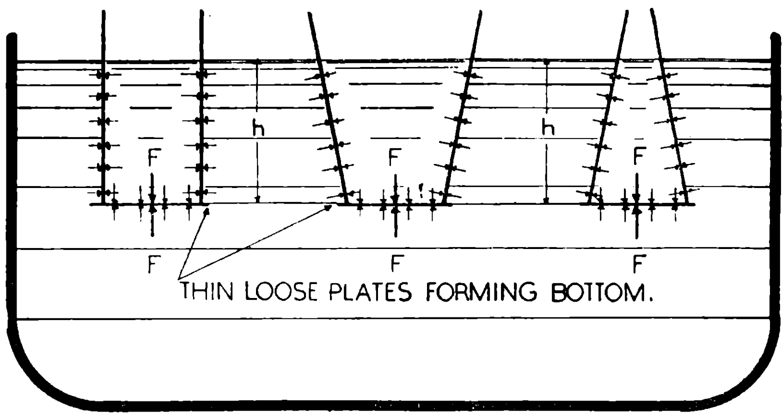


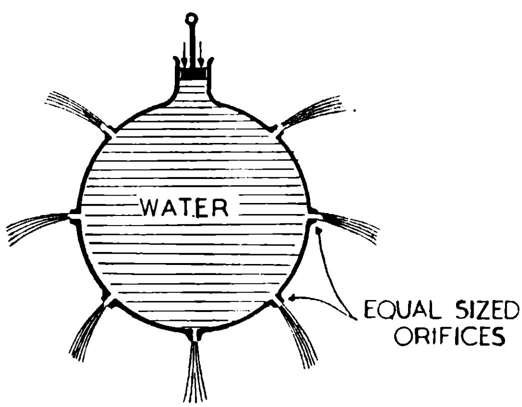
FIG 8

***8. PASCAL'S PRINCIPLE.**

This principle, named after the French philosopher, Pascal. states :—

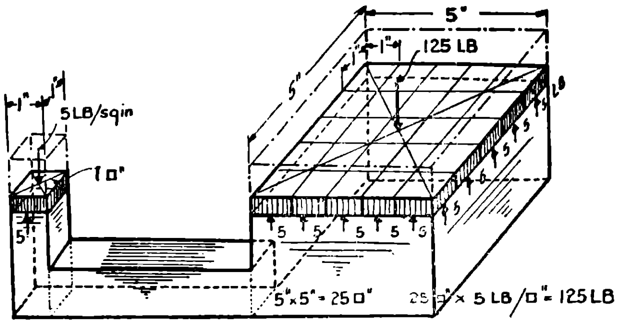
> When pressure is applied at a point to a fluid in a closed vessel, this pressure is transmitted equally and undiminished in all directions throughout the fluid.

Consider the apparatus illustrated in the accompanying diagram. The water spurts uniformly from all the orifices when the piston is pushed down. The pressure exerted on the water by the piston is transmitted equally and undiminished throughout the water.



***8. Pascal's Principle**—*continued.*

The principle has many applications in hydraulic machinery.



The accompanying diagram represents an **hydraulic press.** Suppose the area of the piston *A* is 1 sq. in. and a force of 5 lb. is exerted on it. The pressure exerted by the piston on the liquid in the hydraulic press is 5 lb. per sq. in. This pressure is transmitted undiminished to the second piston *B*.

i.e., the pressure exerted on *B* is 5 lb. **per sq. in.**

If its area is 25 sq. in., the total force exerted on it

$$= 5 \times 25 \text{ lb.}$$

$$= \mathbf{125\ lb.}$$

The larger the ratio of the area of piston *B* to area of piston *A* the greater the force exerted on *B*.

In symbols, if F = the force exerted on the smaller piston
a = the area of the small piston
W = the force exerted on the larger piston
A = the area of the larger piston.

then since the pressures on each piston are the same,

$$\frac{F}{a} = \frac{W}{A}$$

*9. HYDRAULIC SYSTEMS IN AIRCRAFT.

The importance of this lecture will be apparent when the work on hydraulic systems in Aircraft is undertaken in the R.A.A.F. These systems include mechanism to operate flaps, bomb doors, gun turrets, undercarriage, &c. The Gyro pilot and machine gun synchronizing gear are also dependent on hydraulic principles.

EXERCISES.

1. If a force of 75 lb. were applied to a piston of area 15 sq. in., find the pressure exerted on the piston in lb. per sq. in.

2. If a pressure of 14·7 lb. per sq. in. is acting uniformly on a surface of area 100 sq. in., find the total force exerted on the surface.

3. If, in a certain experiment, it was found that a pressure of 16 lb. per sq. inch was acting on the under surface of a flat plate, whereas the pressure on its upper surface was only 13 lb. per sq. in., what was the resultant "lift" on the plate, if its area was 2 sq. ft. ?

4. The average pressure of steam on the piston of a steam-engine was 132 lb. per sq. in. The area of the piston was 48 sq. in. What was the total force on the piston ?

5. Calculate the pressure at the following depths in fresh water :—

12 feet, 100 inches, 100 cm, 10 fathoms, 15 metres.

*6. Find the pressure at the following depths in sea-water, given the density of the sea-water as 1·025 g per cm^3 or 64 lb. per c. ft. :

100 fathoms, 150 metres, 5,280 feet.

*7. In the Pacific Ocean, near to the Phillipine Islands, there is an ocean depth of 32,089 feet. Find the pressure at this depth in lb. per sq. in., using the density of sea-water given in Exercise 6.

*8. Find the pressure at the base of a column of mercury 76 cm. high. (Density of mercury = 13·6 g per cm^3.)

*9. Find the pressure in lb. per sq. in. at the base of a column of mercury 30 inches high. (Specific Gravity of mercury = 13·6.)

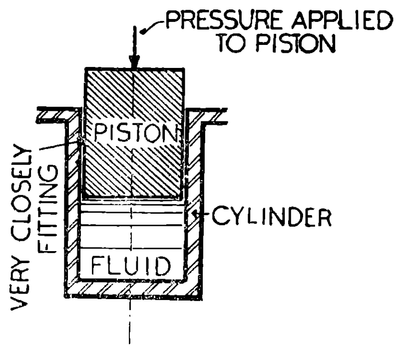
*10. The vertical height of a column of mercury is 29·53 inches. Calculate the pressure, measured in dynes per cm^2, at the base of the column. (1 inch = 2·54 cm.)

ATMOSPHERIC PRESSURE.

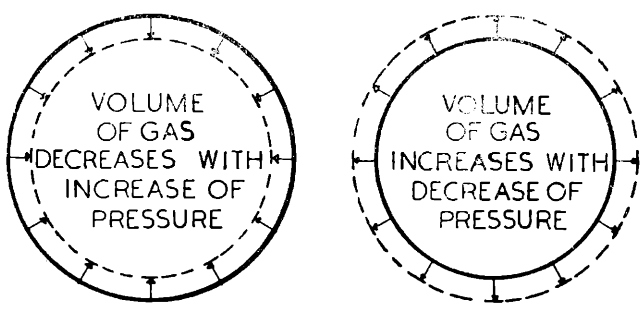
1. The inhabitants on the Earth may be regarded as living at the bottom of an ocean of air, which completely envelopes the globe. The extent of this ocean is not definitely known. The greatest altitude reached by a human being is about 70,000 feet, and this was achieved with the aid of a balloon. By observations on meteors, it is considered that the atmosphere extends upwards to a considerably greater extent. Meteors become luminous due to the friction experienced when travelling through air. The luminosity has been known to develop at an altitude of nearly 200 miles.

2. The atmosphere rotates with the Earth. It is held to the earth's surface by gravity, just as are the waters of the seas. Air therefore has weight and, because of its weight, it is capable of exerting a pressure. The pressure of the air on any surface depends on the weight of the vertical column of air above the surface.

3. Gases are compressible, whereas liquids are almost incompressible. If, in the accompanying diagram, the fluid in the cylinder were a liquid, there would be practically no change in the volume of the fluid even when the pressure exerted on the piston was very large. There would, however, be a considerable change in the volume, if the fluid were a gas.



A given mass of water occupies practically the same volume at great depths as it does near the surface. That is, the density of water may be regarded as uniform throughout. This is not so for



Atmospheric Pressure—*continued.*

air. The pressure on a given mass of air, due to the superincumbent air, will be greater at sea-level than at higher altitudes. The given mass of air will therefore occupy a smaller volume at sea-level than at higher altitudes, i.e., its density will be greatest at sea-level. As a result of variations in the density of air, the atmospheric pressure does not fall off uniformly with altitude. Actually the pressure falls off more rapidly at low altitudes than it does at high.

The illustration in Lecture 7 shows diagrammatically the variation of atmospheric pressure with altitude.

For the first 5,000 feet the pressure drops off 2·46 lb. per sq. inch. For the second 5,000 feet the reduction is 2·13 lb. per sq. inch, whilst for the eighth and ninth 5,000 feet, the reduction is only 0·58 lb. per sq. inch, i.e., the pressure drops off more rapidly at lower altitudes.

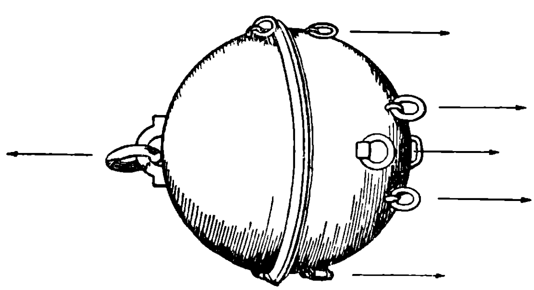
4. At sea-level the average atmospheric pressure is, as indicated in the diagram, about **14·7 lb. per sq. inch.**

Converted to the Metric System this pressure may be expressed as, **1033 g per cm^2.**

5. The air pressure at any locality varies from day to day. These variations and their measurement will be discussed in the next lecture.

***6. Some effects of air pressure.**

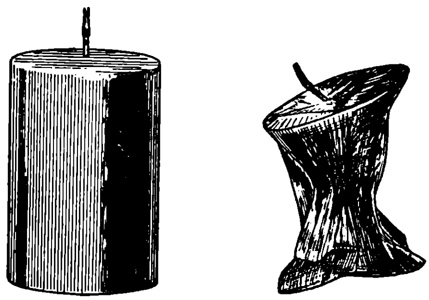
(1) One of the earliest experiments to prove that air exerted a pressure was performed in the 17th Century at Magdeburg (Prussia). Air was removed from between two hollow hemispheres fitting closely at their rims. It was found that eight horses attached to each hemisphere were required to drag them apart.



(2) A simple experiment can be performed in the laboratory to show the effect of air pressure. The accompanying illustration represents a thin-walled metal vessel, open to the air only at the small hole shown. If water were boiled in the vessel for a few

Atmospheric Pressure—*continued.*

minutes, the steam generated would displace the air inside. If, after removing the flame and closing the hole, the vessel were cooled by running cold water over it, the steam within would condense,



producing a region of reduced pressure. The greater pressure of the air on the outside would crush the vessel, if a suitable vessel, such as a kerosene tin, were chosen.

(3) A question often arising is that which asks why a human-being does not suffer discomfort because of the considerable total force, due to air pressure, that is acting on the body. The explanation is that the body has always been subjected to the pressure of the air and has developed accordingly, in order to withstand the load imposed on it. Discomfort arises rather when the pressure is reduced, as in the case of aviators flying at high altitudes. The blood of a human-being contains dissolved air. Just as the dissolved gases in aerated waters are liberated when the pressure is reduced (as on opening the cork of the bottle), so the gases dissolved in the blood are liberated as the pressure on the body is reduced. Normally, the walls of the blood vessels are subjected to equal pressures within and without, the pressure of the blood and its dissolved gases being balanced by the external atmospheric pressure. If the external pressure is greatly reduced, and particularly if the reduction is rapid, bleeding may occur through thin-walled vessels, as in the nose and ears.

(4) It is for reasons similar to these that men who have to work in compressed air are restored to normal atmospheric pressure gradually. Serious discomfort, insensibility and even death might otherwise result. (N.B.—The pressure changes in the case of such workmen are usually much greater than can ever occur in the case of a pilot.)

EXERCISES.

1. What would be the total force on a surface of area 100 sq. in., (*a*) at sea-level, (*b*) at 15,000 feet, (*c*) at 30,000 feet ?

2. What would be the pressure in g per cm^2 exerted on a surface of area 100 cm^2 at the same altitudes as in Exercise (1) ? Convert each value to dynes per cm^2.

*3. Tabulate the reductions in the pressure of the atmosphere for every 5,000 feet, from sea-level to 45,000 feet.

4. What is the difference in air pressure between one locality at an altitude of 5,000 feet, and another at an altitude of 20,000 feet ?

*5. What is the air pressure, expressed in g per cm^2, at 35,000 feet ? (Use conversion tables from Arithmetic Course.)

*6. Why does the liquid rise in an ordinary drinking-straw when being used for drinking ?

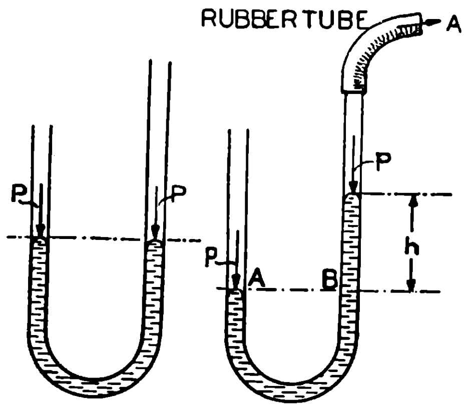
*7. Balloons are not filled to their utmost capacity on their release. Why ?

THE BAROMETER AND ALTIMETER.

1. THE MERCURY BAROMETER.

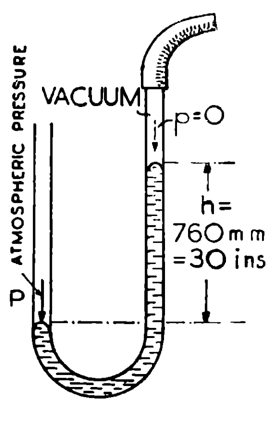
The Barometer is an instrument for measuring the pressure of the atmosphere.

Consider a glass tube, formed into the shape of a letter U, as shown in the diagram. If a liquid, say mercury, is poured into the tube, it will stand at the same height in each arm of the tube on coming to rest. The pressure acting on the surface of the mercury in each arm is equal to the atmospheric pressure (represented by P in the diagram). Suppose air is removed from one arm. The pressure in that arm will be reduced and will be less than the atmospheric pressure in the other. Consequently the mercury will rise in the arm in which there is reduced pressure.



Now, the atmospheric pressure (P) in the one arm = the reduced pressure (p) + the pressure at B, in the other arm, due to the column of mercury above B.

Should air still be removed from the arm concerned, the value of p will be further reduced and the mercury will rise higher in the arm. If the value of p were eventually reduced to zero, the atmospheric pressure (P) would be equivalent only to the pressure due to the column of mercury, "h" units high.

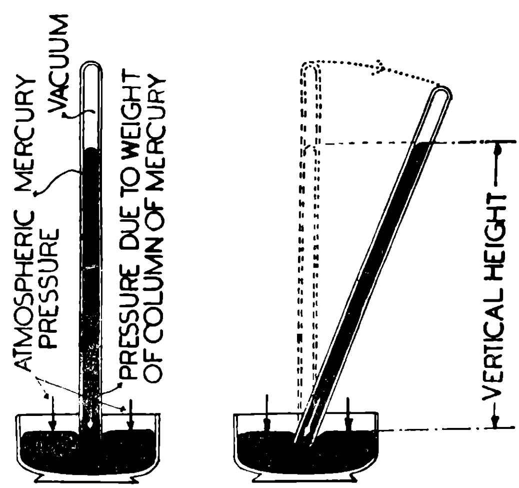


1. **The Mercury Barometer**—*continued.*

For an atmospheric pressure of 14·7 lb. per sq. in., the height (h) of the column of mercury concerned is **760 mm,** or approximately **30 inches.**

The atmospheric pressure varies from day to day, and the height of the column of mercury correspondingly fluctuates. The height of the column can be regarded as a measure of the atmospheric pressure on any particular occasion. This is the **principle of the mercury barometer.**

A simple type of mercury barometer can be made by completely filling, with mercury, a glass-tube, approximately a yard long, and closed at one end, and then inverting it over another vessel of mercury as shown in the diagram. The open end of the tube is kept sealed by a thumb whilst being inverted and placed in the mercury in the other vessel.



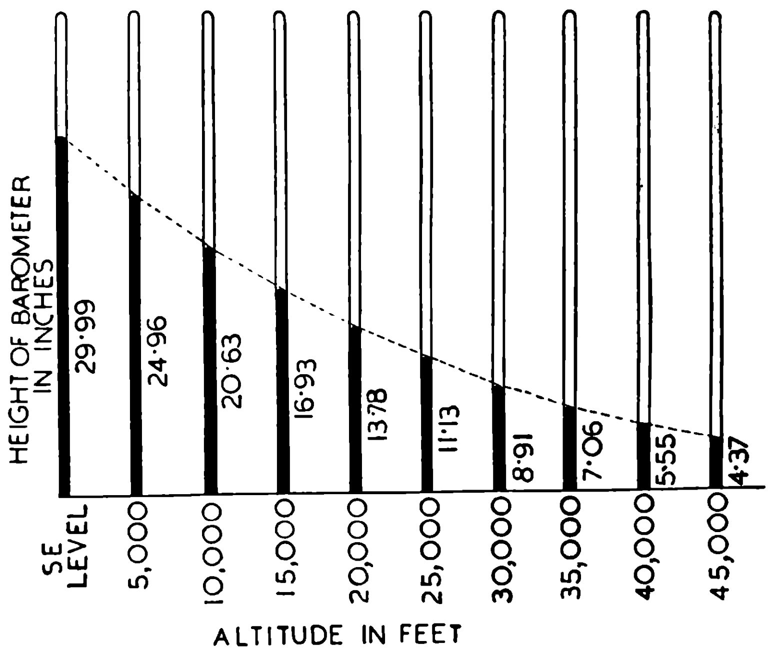
The mercury in the tube will fall until it stands at a constant height above the level of the mercury in the vessel outside, the space above the column of mercury being a partial vacuum, containing only mercury vapour.

This height can be read, in either inches or millimetres, and taken as a measure of the atmospheric pressure on that occasion.

N.B.—When speaking of the height of a barometer, the **vertical** height must be understood. The vertical height of the mercury remains constant, even when the barometer tube is tilted.

2. THE ALTIMETER.

It has been pointed out that the pressure of the atmosphere decreases with altitude. The reading on a barometer consequently becomes less as it is taken to higher altitudes. These variations in the readings of a barometer provide a means of ascertaining the height of a place above sea-level, so long as the rate at which the pressure readings vary with altitude is known. The following diagram represents barometric readings, in terms of inches of mercury, at various altitudes :—



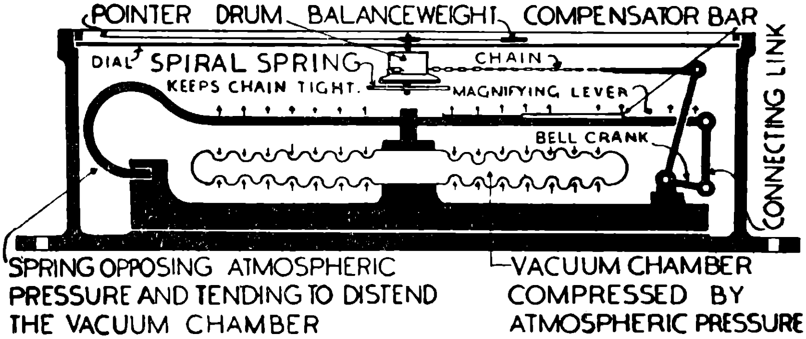
This diagram may be compared with similar diagrams in Lecture 8, 1A/2b/1 Arithmetic and in Lecture 7, 2/2b/1 Science.

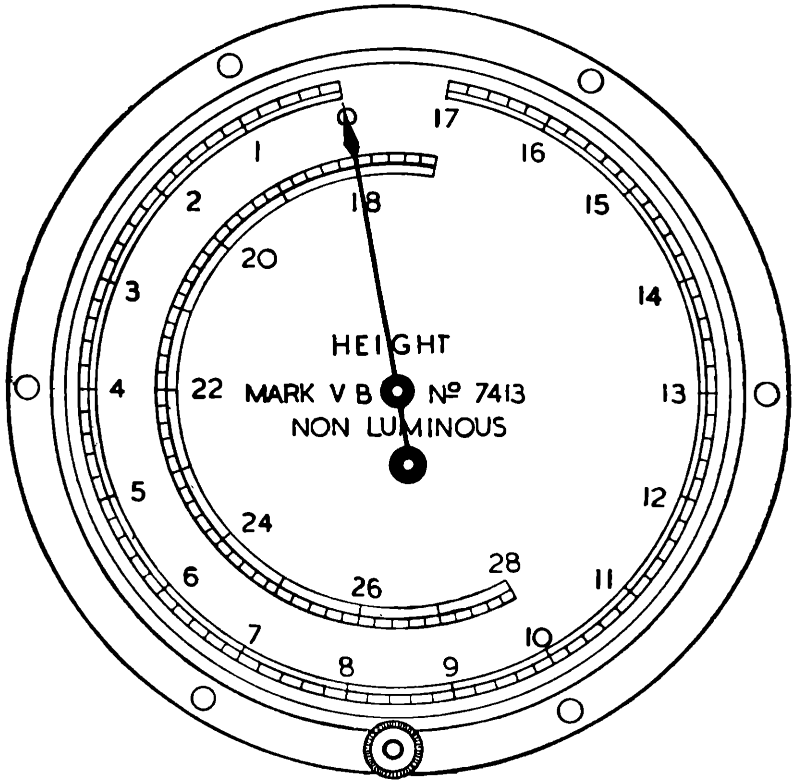
It is inconvenient to transport a mercury barometer for the purpose of measuring altitudes. The **aneroid barometer** however is a type of barometer in which no liquid is used. (The word "aneroid" incidentally is derived from Greek words meaning "not moist".) Its structure is sufficiently compact to render it readily transportable. Indeed, models are made of a size suitable for carrying in one's pocket. The aneroid barometer is consequently very much more suitable for altitude work than the mercury type. Its scale is graduated to read pressure in terms of inches or millimetres of mercury. A second scale may be calibrated to read altitudes.

(**N.B.**—The mercury barometer is used as the standard for the calibration of aneroid barometers.)

2. **Altimeter**—*continued.*

The **altimeter**, as used in aircraft, is simply an aneroid barometer, the scale indicating the altitude of the aircraft by direct reading.





The principal parts of the instrument are as illustrated. It consists, as shown, of a sealed shallow circular metal box, from which most of the air has been removed, and which is fixed, at the

2. **Altimeter**—*continued.*

bottom, to the base of the instrument. The lid of the box is corrugated and, being specially elastic, is sensitive to small variations of atmospheric pressure. A strong broad spring, attached at one end to the base of the altimeter, is connected at the other to the centre of the lid of the box. A light thin rod connects this upper end of the spring to a system of levers, which in turn is connected by means of a fine flexible chain to a pointer. The pointer moves over the graduated scale of the altimeter (see accompanying diagram).

A decrease of atmospheric pressure reduces the load on the box, whilst an increase in pressure causes an increase in the load. The box thus expands and contracts. The movements of the lid of the box are transmitted, through the spring, to the system of levers. The purpose of the system of levers is to magnify the movements.

***3. ALTIMETER ADJUSTMENTS.**

(1) The bar compensating for temperature changes is usually made by soldering a strip of steel to the brass lever. Steel and brass expand differently under the influence of heat. Consequently the connecting lever bends with a change of temperature. The amount of bending is such as to compensate for the effects produced, in the metal box and the spring, by changes of temperature.

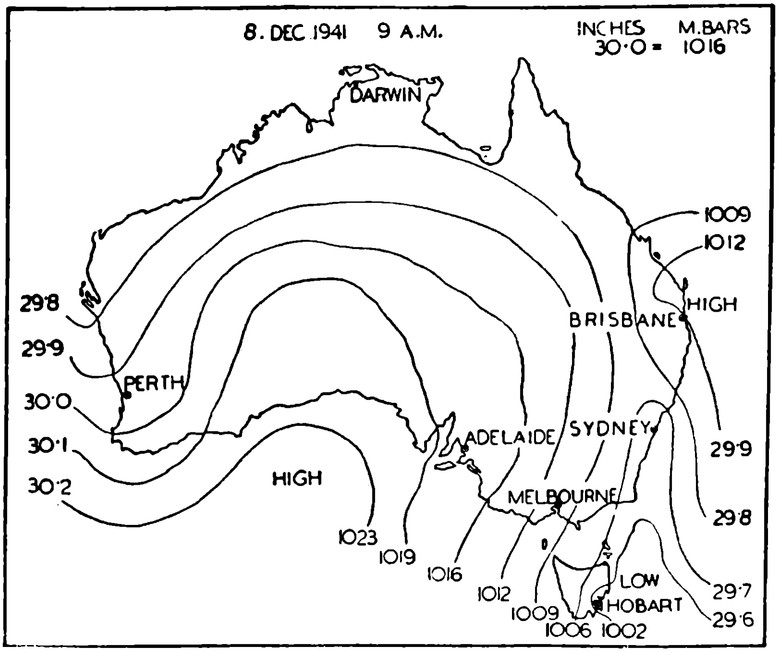
(2) An adjustment is usually provided by which the pilot can set the reading of the altimeter in accordance with the ground atmospheric pressure. The adjustment is effected by changing the relative positions of the scale and pointer. A knob is shown at the base of the altimeter dial illustrated. By means of this knob, the scale can be rotated.

In general the altimeter will indicate the height of the aeroplane at any moment; but the point above which the height is measured will depend on the manner in which the altimeter is set. If the dial is adjusted, before taking off, so that the pointer reads zero, the altimeter will indicate, during flight, the **height above the home aerodrome,** and provided the atmospheric pressure at the aerodrome does not change during the flight the reading will return to zero on landing at the same aerodrome. Actually, due to the imperfect elasticity of the diaphragm box and spring, the pointer may not return to its original position at once. No altimeter is free of this "lag". After a rapid descent, say of 20,000 feet, the altimeter may read 300 feet instead of zero. A similar lag occurs when the aircraft ascends rapidly. If the **height above sea-level** is required during flight, the altimeter must be set, before taking off, so that the pointer indicates the height of the aerodrome above sea-level. If this is done, the height of the aircraft above sea-level will be shown at any moment during the flight, provided the conditions, under which the instrument was originally set, have not changed in the meantime.

To obtain more accurate determinations of height, corrections should be made for temperature, which influences the atmospheric pressure. Computors are available for doing this.

*4. THE MILLIBAR.

A weather map has a number of curved lines drawn across it, which are called **isobars.** These lines join places on the map at which, from information received, the pressures are known to be equal at a specified time.



Two values are affixed to each isobar, one at each end. The smaller number (e.g. 30·0) indicates the height of the barometer in inches, corresponding to the particular atmospheric pressure. The other value (e.g. 1016) indicates the air pressure measured in **millibars.**

An atmospheric pressure of 1,000,000 dynes per sq. cm is called **a bar.** This pressure is equivalent to that due to a column of mercury

750 mm (or **29·53 inches**) high.

Check :

The pressure at the base of a column of mercury 750 mm high

$$= 75 \times 13{\cdot}6 \text{ g per cm}^2,$$
$$= 75 \times 13{\cdot}6 \times 981 \text{ dynes per cm}^2,$$
$$= \mathbf{1{,}000{,}000 \text{ dynes per cm}^2}.$$

***4. The Millibar**—*continued.*

The **Millibar** is one-thousandth of a bar and so represents a pressure of **1,000 dynes per cm^2.**

This pressure is equivalent to that at the base of a column of mercury 0·02953 inch high.

Since, the pressure represented by 0·02953 inch mercury is 1 millibar

the pressure represented by 1 inch is $\frac{1}{0 \cdot 02953}$ millibars

$=$ **33·87 millibars.**

Summarizing,

1 millibar is equivalent to a barometric reading of 0·02953 inches.

33·87 millibars is equivalent to a barometric reading of 1 inch.

EXAMPLES.

(1) How many millibars are equivalent to the pressure at the base of a column of mercury **1 cm** high ?

1 inch (or 2·54 cm) of mercury represents a pressure of 33·87 millibars

i.e. 1 cm of mercury represents a pressure of $\frac{33 \cdot 87}{2 \cdot 54}$ millibars

$=$ **13·34 millibars.**

(2) How many millibars are represented by a column of mercury 30 inches high ?

1 inch of mercury represents 33·87 millibars

30 inches of mercury represents 33·87 $\times$ 30 millibars

$=$ 1016·1 millibars

$=$ **1016 millibars,** approx.

(Check this value on the weather map supplied.)

EXERCISES.

1. Give reasons why a mercury barometer would not be suitable to give heights of aircraft.

2. Suppose an aneroid barometer read 22·8 inches on the slope of a mountain. Using the scale provided above, what would be the approximate value for the altitude above sea-level ?

Exercises—*continued.*

3. From the scale provided, construct a graph to show heights of mercury in a barometer for altitudes above sea-level.

From your graph read off :—

(1) altitudes at which height of mercury is 8·4 in., 15 in., 22·5 in.

(2) heights of mercury at altitudes of 12,000 feet, 27,500 feet, 3,000 feet.

*4. Convert, to millibars, the barometric pressures, given in the diagram above, for altitudes from sea-level to 45,000 feet.

*5. An aneroid barometer is furnished with the following chart, indicating the weather corresponding to various barometric readings :—

Below 29 in.	Stormy
29 in.–29·5 in.	Rain
29·5 in.–30 in.	Change
Above 30 in.	Fine.

How would you interpret the following readings ?

(*a*) 1008 millibars.

(*b*) 770 millimetres.

(*c*) 970 millibars.

(*d*) 747 millimetres.

(*e*) 987 millibars.

(*f*) 739 millimetres.

*6. A barometer read on consecutive days 29·6″, 29·86″, 30·17″. The readings from the meteorological station on the same days were 1002·3, 1013·6, 1024 millibars.

Find for each day the error of the instrument, in millibars.